Lagrangian maps as a new tool to simulate transport processes in the ocean

S.V. Prants, M.V. Budyansky, M.Yu. Uleysky

Laboratory of Nonlinear Dynamical Systems
Pacific Oceanological Institute of the Russian Academy of Sciences,
Vladivostok, Russia, URL: dynalab.poi.dvo.ru

PICES, Vladivostok, 28 September, 2017

Contents

Lagrangian approach

The development of the Lagrangian methods to study transport and mixing was advanced due to several factors:
(i) Satellite monitoring of the ocean and atmosphere provide continuous, near real-time and global data at high space resolution.
(ii) Satellite-tracked buoys and drifters provide real-time information about ocean circulation in a high-frequency manner.
(iii) The development of high-resolution global and regional numerical models of ocean circulation allows to simulate transport and mixing at sub- and mesoscale.
(iv) Ideas and methods from dynamical systems and chaos theory. In complex systems it is much more productive not to search for specific solutions (trajectories) of governing equations but to find geometric templates (manifolds) in the phase space governing transport and mixing.

Advection equations

The Lagrangian analysis of large-scale transport and mixing in the ocean is based on solving equations of motion for a large number of synthetic tracers advected by a satellite-derived velocity field (`http://www.aviso.altimetry.fr`) archived daily on a $1/4^\circ \times 1/4^\circ$ grid covered all the global ocean or advected by a numerically generated velocity field

$$\dot{\lambda} = u(\lambda, \phi, t), \qquad \dot{\phi} = v(\lambda, \phi, t), \tag{1}$$

where u and v are angular zonal and meridional velocities on the Earth sphere, ϕ and λ are latitude and longitude, respectively.
Bicubical spatial interpolation and third order Lagrangian polynomials in time are used to provide accurate numerical results. Lagrangian trajectories are computed by integrating the advection equations (1) with a fourth-order Runge-Kutta scheme.

1. Fields of Lagrangian indicators

Lagrangian indicator is a quantity that is a function of fluid particle's trajectory.
Examples: a distance passed by a particle, its absolute, zonal and meridional displacements, a distance between initially close particles for a given time, number of cyclonic and anticyclonic particle's rotations, entrance, residence and exit times, the geographic areas where particles came from to a study area, etc.
Lagrangian indicators gives an information on the origin, history and fate of fluid particles.
In order to display the enormous amount of information, we compute Lagrangian maps which are plots of Lagrangian indicators versus particle's initial positions.

See the book: S.V. Prants, M.Yu. Uleysky, M.V. Budyansky. Lagrangian oceanography: large-scale transport and mixing in the ocean. Berlin, New York. Springer. 2017. 271 p.

Lagrangian maps

A study area or a material line are seeded with a large number of synthetic tracers whose trajectories are computed forward or backward in time for a given period of time.

When integrating advection equations forward in time one computes particle's trajectories in order to know their fate.

When integrating them backward in time one gets information on their origin and history.

Integrating advection equations, we calculate a specific Lagrangian indicator for each fluid particle and code its values by color on a geographic map on a fixed date. Computing that Lagrangian indicator for a large number of particles, we get the corresponding Lagrangian map which shows the oceanographic situation in the study area on a given day in the field of that indicator.

Specific Lagrangian indicators

The finite-time displacement D is a distance between the final, (λ_f, ϕ_f), and initial, (λ_0, ϕ_0), positions of advected particles on the Earth sphere

$$D \equiv R_E \arccos[\sin\phi_0 \sin\phi_f + \cos\phi_0 \cos\phi_f \cos(\lambda_f - \lambda_0)]. \tag{2}$$

[SP et al. Ocean Modelling v.38, p.114, 2011].

The Lagrangian indicator L is a measure of a distance in geographic minutes passed by advected particles. It is computed backward/forward in time for a period of time T, and its value is coded by a gradation of the grey color

$$L = \int_0^T \sqrt{u^2 + v^2} dt \tag{3}$$

[SP et al. Ocean Modelling 2017, in press].

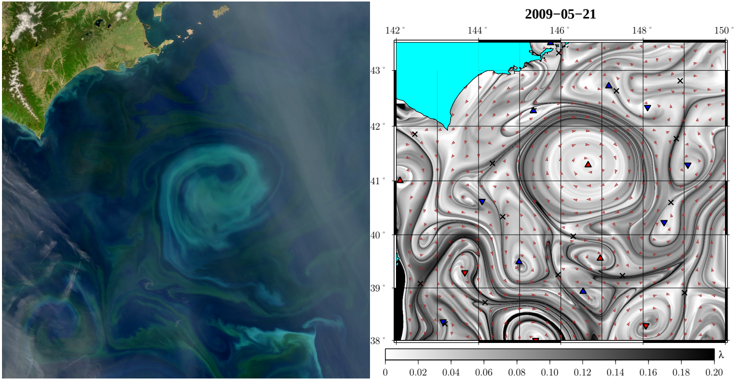
L maps deliniate a boundary between the core and periphery of eddies and a history and fate of water masses involved in the vortex motion.

The finite-time Lyapunov exponents (FTLE) are computed backward/forward in time

$$\Lambda(t, t_0) = \frac{\ln \sigma(t, t_0)}{t - t_0}, \tag{4}$$

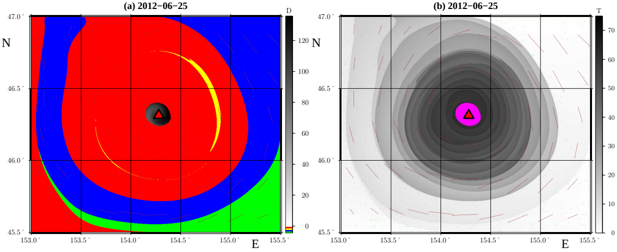
the ratio of the logarithm of the maximal possible stretching in a given direction to the integration time interval $t - t_0$. $\sigma(t, t_0)$ is a maximal singular value of the evolution matrix. $\Lambda_\pm$s deliniate repelling (**divergency**) and attracting (**convergency**) manifolds of hyperbolic trajectories and organize a flow geometry in the study area [SP, Uleysky, Busyansky. Ocean Modelling v.38, p.114, 2011].
Stable (repelling) W_{s} and unstable (attracting) W_{u} manifolds of a hyperbolic trajectory $\gamma(t)$ are material lines consisting of a set of points through which at time moment t pass trajectories asymptotical to $\gamma(t)$ at $t \to \infty$ (W_{s}) and $t \to -\infty$ (W_{u}).

2. Eddies in the fields of Lagrangian indicators



Satellite image of a Hokkaido eddy. Altimetry-based simulation in the fields of forward- and backward-in-time Lyapunov exponents shows much finer features with a shielded anticyclone surrounded by small cyclones. Crosses (circles) are hyperbolic (elliptic) stagnation points with zero velocity.

Anatomy and age of eddies

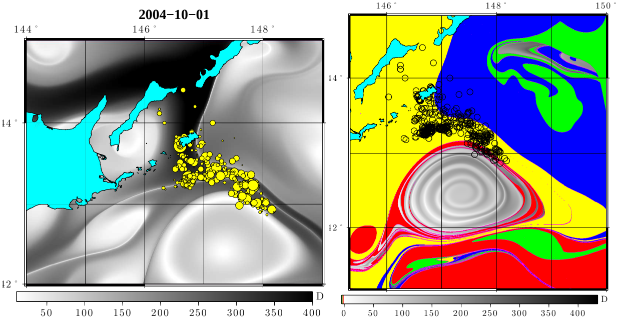


a) The origin map shows from which surface water masses a studied eddy consists of: where waters of the sampled Kuril eddy came from for 2.5 months in the past: yellow — from the west, green — the east, blue — the north, red — the south, grey — the water that has been in the eddy core for 2.5 past months.

b) The entrance-time map shows "age" of different parts of the eddy. Each vortex ring corresponds to the day T, when the corresponding water parcels have been captured by the eddy. The darker the color the earlier the corresponding water has been trapped.

3. Lagrangian fronts

1. Common hydrological front is a boundary between waters with strongly distinct physical properties (temperature, salinity, density etc.).
2. Each fluid parcel can be attributed in addition different kinds of Lagrangian indicators.
3. A line or a surface with local maxima of the gradient of a Lagrangian indicator deliniates the corresponding Lagrangian front.
4. Lagrangian fronts provide a valuable information about origin and history of water masses. "Favorable" Lagrangian fronts have been shown to attract schools of fish [S.V. Prants, M.V. Budyansky, M.Yu. Uleysky. Identifying Lagrangian fronts with favourable fishery conditions. Deep Sea Research I. V. 90, p.27-35 (2014)]

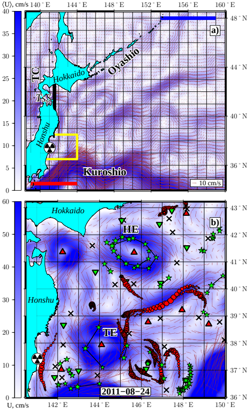


Drift (left) and origin (right) maps near the Hokkaido and Kuril islands on 1 Oct 2004. Lagrangian fronts between subarctic (blue and green) and transformed subtropical (yellow and red) waters. The maps delineate fronts between distinct water masses around the Hokkaido eddy coming from the Okhotsk Sea, Kuroshio area, open ocean and Oyashio for half a year. Saury catching places accumulate near the strongest Lagrangian fronts with the maximal gradient of the displacement of particles [SP et al Deep Sea Res.I. V.90 p.27 2014].

4. Tracking for eddies with a high risk to be contaminated by Fukushima-derived radionuclides in 2011 and 2012

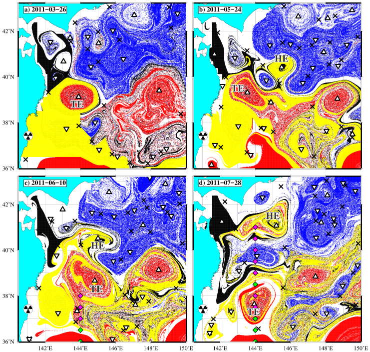
After the Fukushima accident in March 2011, it was important not only to find the pathways by which the radioactive water spread in the ocean but to track and document as well how radionuclides were gained, retained and released by different eddies in the vast area of the Kuroshio–Oyashio frontal zone.
S.V. Prants, M.V. Budyansky, M.Yu. Uleysky. Lagrangian simulation and tracking of the mesoscale eddies contaminated by Fukushima-derived radionuclides. Ocean Science. V.13 P.453-463 (2017).

Altimetry-based Lagrangian origin (O) maps show and document the origin of water masses inside each mesoscale eddy and around it for the whole period of their life.



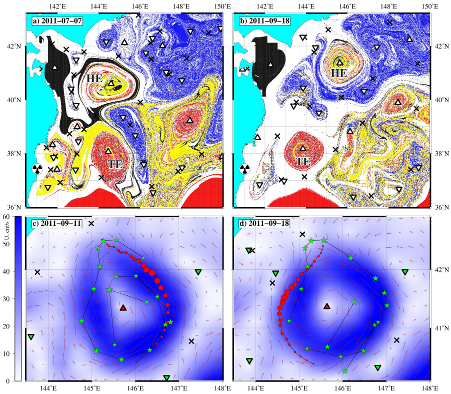
a) Mean (1993–2016) AVISO field, b) on 24 Aug 2011. All the area was seeded with a half a million particles whose trajectories were computed backward in time for 2 years to fix where they came from: yellow from the Fukushima area, red (Kuroshio), blue (Oyashio), black (Tsugarru Current) and gree (Okhotsk Sea).

O maps with imposed sampling stations in June-July 2011



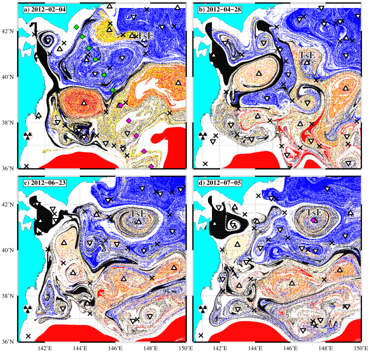
Yellow water came from Fukushima area. Red, black, blue ones – from Kuroshio, Tsugaru, Oyashio currents. Tohoku eddy was sampled in June, July [Buesseller 2012, Kaeriyama 2013] to be strongly contaminated in corresp. with O map. Hokkaido eddy was measured to be strongly contaminated in corresp. with O map. Diamonds: measured background (green) and much higher (magenta) Cs conc.

Contaminated Hokkaido eddy partly sampled in July 2011



The Hokkaido eddy was estimated to be strongly contaminated even a year after the accident. It was partly sampled in July 2011 to be strongly contaminated in correspondence with the O map. a), b) O maps in July–Sept. 2011. c), d) Track of a drifter (red circles) captured by this eddy [SP et al, Ocean Sci V.13, 453, 2017].

O maps with sampling stations in Feb and July 2012



a) Measured background (green) and increased (magenta) Cs conc. in Feb 2012 along the shown line [Kumamoto 2014] in corresp. with O map: low conc. in Oyashio blue water, increased in Fukushima yellow one. b)-d) Tsugaru eddy was born after splitting of the contaminated Hokkaido eddy and measured to have increased Cs conc. even in July 2012 [Budyansky et al, DSR I V.96, 15, 2015].

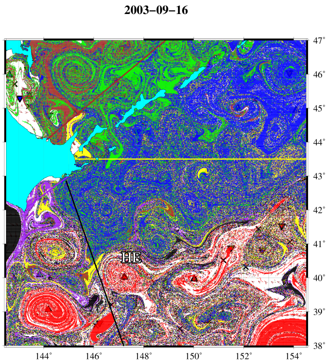
5. Lagrangian documentation of eddy's life

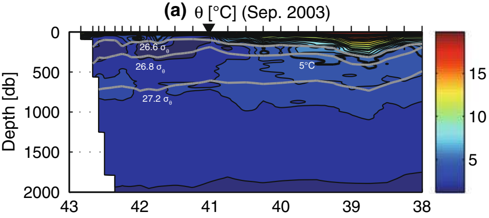
To verify our methodology to track the origin of water masses in the core and at periphery of mesoscale eddies, we need to compare simulation results with observation ones.

We focus now at the case study of a Hokkaido eddy which has been carefully sampled in 2003 and 2004 by Sachihiko Itoh, Yugo Shimizu, Shin-ichi Ito, Ichiro Yasuda J. Ocean. V.67 P.281 2011.

The area was seeded with a half a million particles whose trajectories were computed backward in time for 2 years to fix where they came from: red from the Kuroshio, blue (Oyashio), black (Tsugarru Current), green and brown (Okhotsk Sea).

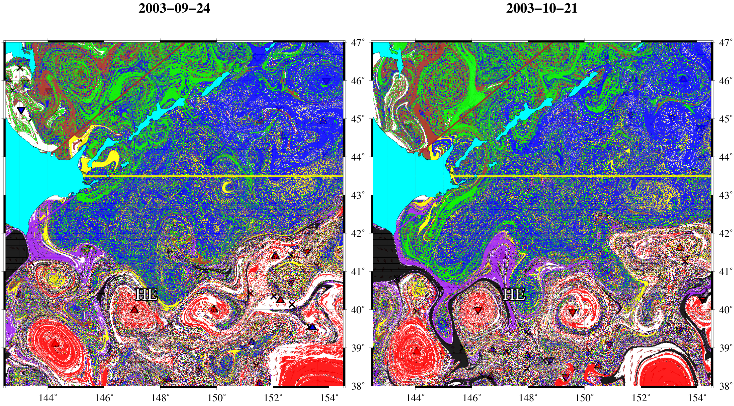
Comparison with a transect along the A line in Sept 2003





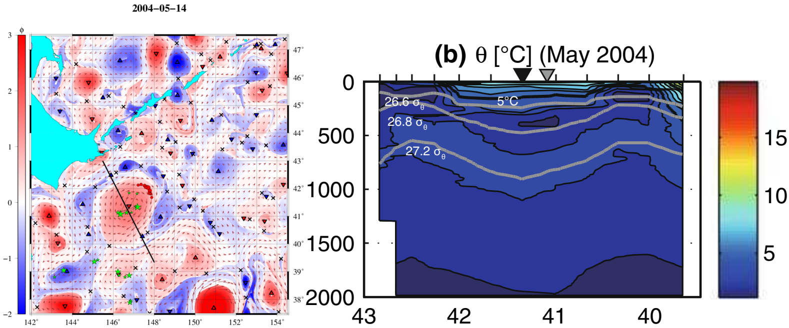
Red waters came from the Kuroshio, blue (from Oyashio), black (from Tsugaru Current), green and brown (from Okhotsk Sea). O map on 16 Sept 2003 and potential temperature along the A line [Itoh 2011] that did not cross the Hokkaido ACE but it crossed a smaller ACE with the center at (38.7N, 146.2E) which is clearly seen on the O map and in the cross section.

Entrainment event of Oyashio water in Sept–Oct 2003



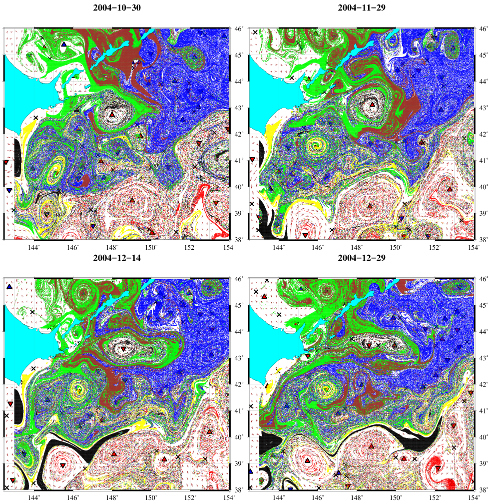
"On 24 Sept the eddy began to entrain the cold Oyashio water together with the two floats into its core. The entrainment event was completed by 22 October, when the surface water had cooled by about 4C" [Itoh 2011]. Look at the intrusion of the violet water.

Comparison with a transect along the JMA line in May 2004



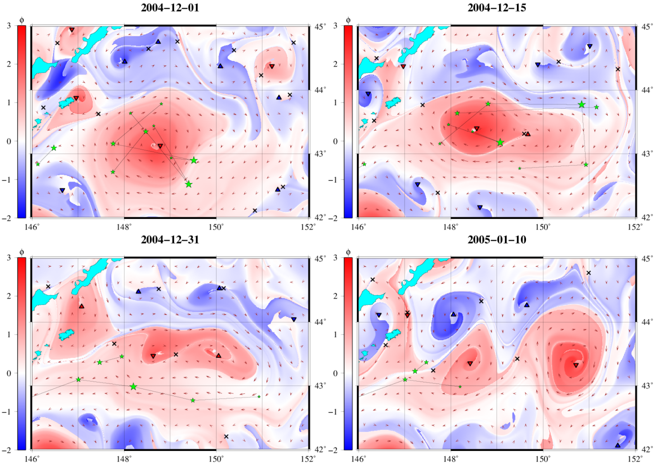
Map of anticyclonic (red) and cyclonic (blue) rotations of particles on 14 May 2004 (for two weeks in the past) and potential temperature along the JMA line [Itoh 2011]. The line crossed the Hokkaido shielded eddy and two divergency areas with elliptic points at 42.5N, 146E and 40.2N, 147.5E. The features are seen on the map and in the cross section with corresponding depression and elevations of the isopycnal surfaces.

Entrainment event of Okhotsk Sea water in Nov–Dec 2004



The O maps document a winding of the intrusion of “green” water from the Okhotsk Sea to the eddy in the late fall and winter of 2004.

Deformation and splitting of the eddy



“Prominent interaction, observed between the eddy and water from the Okhotsk Sea in Dec 2004, caused its marked elongation and detrainment of the floats” [Itoh et al. J. Ocean. 2011].

How reliable is Lagrangian simulaion with AVISO field?

It is impossible to compute exactly trajectories of specified particles in an imperfect AVISO field. Even if it was absolutely perfect, it would be impossible to do that because of exponential sensitivity to small variations in initial conditions and parameters in a turbulent velocity field in the real ocean.

So, what we compute? We compute trajectories for a large number of particles and pathways for large water masses. How well the computed pathways do correlate with the real (“true”) ones?

There exists the “shadowing lemma” which states that every numerically computed trajectory stays uniformly close to some true trajectory (with slightly altered initial position), in other words, a computed trajectory is “shadowed” by a “true” one. We compute not “a true” trajectory but the one which is close to this “true” trajectory. If it is done with a large number of particles, computed pathways for water masses are expected to follow real ones (besides some regions close to the coast where the AVISO field is absent).

How robust are Lagrangian structures to errors in AVISO

A few numerical experiments have been carried out to test the sensitivity of strong LCS to errors in velocity fields in different basins. They were found to be relatively insensitive to both sparse spatial and temporal resolution and to interpolation method (Haller 2002, Harrison 2010, Keating 2011, Hernandez 2011, Cotte 2012).

- 0 Dim stationary points. Long-lived elliptic points in the centers of mesoscale eddies and hyperbolic points around them are computed reliably because the AVISO field is smooth around them. The accuracy in fixing of eddy's centers has been found to be 5–7 km for *in situ* sampled eddies.
- 1Dim Lagrangian fronts and LCS. They are robust to errors in velocity field if they are strongly attracting or repelling and exist for a sufficient long time (Haller 2002, SP et al 2014). Though simulated trajectories diverge exponentially from "true" trajectories nearby them, the very LFs and LCSs are not expected to be perturbed to the same degree, because errors in the trajectories spread along them.

How robust are Lagrangian structures to errors in AVISO

- 2Dim cores of mesoscale eddies and intrusions. Such features are computed reliably if they formed and propagated as “coherent” large-scale water masses.
- 2Dim submesoscale features. The altimetry-based Lagrangian maps were shown to reproduce even some submesoscale features if they appear due to evolution of the mesoscale 2Dim advection field (lobes, swirls, small filaments) not as a result of local frontogenesis or ageostrophic instabilities. The simulation results were compared with SST and color images in different basins (Lehahn et al 2007, SP et al 2014–2017).

The future altimetry mission SWOT will improve resolution of 2D maps of SSH and accuracy of representation of coastal circulation structures and their temporal evolution. It would allow to improve greatly accuracy of our Lagrangian maps.
The ultimate tests are comparisons of simulation results with observation ones.

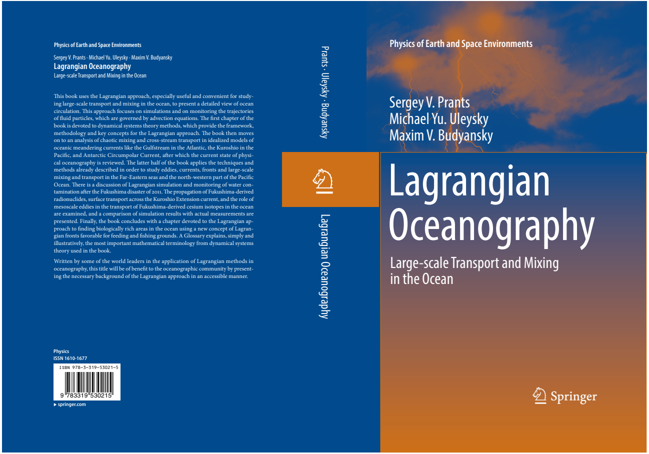
How to use Lagrangian maps in the R/V cruises

- Before planning R/V cruises, it is instructive to compute altimetry-based Lagrangian maps of different kind to know oceanographic situation in the study area: location, type and properties of eddies, currents, streamers and streaks in the area. It helps to plan the tasks of the cruise.
- The real-time altimetry-based Lagrangian maps sent to a board allow to save fuel and optimize the vessel's route.
- Lagrangian maps sent to a board allow to fix eddy's centers (elliptic points) in order to optimize cross section locations.
- Lagrangian maps track changes in the oceanographic situation in the study area practically in the real time and allow to adjust a cruise program.

Conclusion

The Lagrangian maps, computed with altimetric or numerically-derived velocity fields, seem to be more effective to simulate transport and mixing in the ocean and to identify and track eddies than commonly used techniques because the Lagrangian maps are imprints of history of water masses, they are not "instantaneous" snapshots. That is why one could "see" Lagrangian fronts and eddies and document their transformations more accurately on Lagrangian maps.
We demonstrate in this talk how our methodology works with the altimetric velocity field and mesoscale features.
We verified the methodology by comparing the simulation results with measurements of concentration of Fukushima-derived radioisotopes in eddies and with sampling of Hokkaido eddies.
Support from the Russian Science Foundation (16-17-10025) and Russian Foundation for Basic Research (16-05-00213).

The book now issued by Springer at
http://www.springer.com/gp/book/9783319530215



Physics of Earth and Space Environments

Sergey V. Prants · Michael Yu. Uleysky · Maxim V. Budyansky
Lagrangian Oceanography
Large-scale Transport and Mixing in the Ocean

This book uses the Lagrangian approach, especially useful and convenient for studying large-scale transport and mixing in the ocean, to present a detailed view of ocean circulation. This approach focuses on simulations and on monitoring the trajectories of fluid particles, which are governed by advection equations. The first chapter of the book is devoted to dynamical systems theory methods, which provide the framework, methodology and key concepts for the Lagrangian approach. The book then moves on to an analysis of chaotic mixing and cross-stream transport in idealized models of oceanic meandering currents like the Gulfstream in the Atlantic, the Kuroshio in the Pacific, and Antarctic Circumpolar Current, after which the current state of physical oceanography is reviewed. The latter half of the book applies the techniques and methods already described in order to study eddies, currents, fronts and large-scale mixing and transport in the Far-Eastern seas and the north-western part of the Pacific Ocean. There is a discussion of Lagrangian simulation and monitoring of water contamination after the Fukushima disaster of 2011. The propagation of Fukushima-derived radionuclides, surface transport across the Kuroshio Extension current, and the role of mesoscale eddies in the transport of Fukushima-derived cesium isotopes in the ocean are examined, and a comparison of simulation results with actual measurements are presented. Finally, the book concludes with a chapter devoted to the Lagrangian approach to finding biologically rich areas in the ocean using a new concept of Lagrangian fronts favorable for feeding and fishing grounds. A Glossary explains, simply and illustratively, the most important mathematical terminology from dynamical systems theory used in the book.

Written by some of the world leaders in the application of Lagrangian methods in oceanography, this title will be of benefit to the oceanographic community by presenting the necessary background of the Lagrangian approach in an accessible manner.

Physics
ISSN 1610-1677
ISBN 978-3-319-53021-5
9 783319 530215
▶ springer.com

Prants · Uleysky · Budyansky

Lagrangian Oceanography

Physics of Earth and Space Environments

Sergey V. Prants
Michael Yu. Uleysky
Maxim V. Budyansky

Lagrangian Oceanography

Large-scale Transport and Mixing
in the Ocean

Springer