Optimal harvest strategies of sandfish based on a stage-structured model in the East Sea

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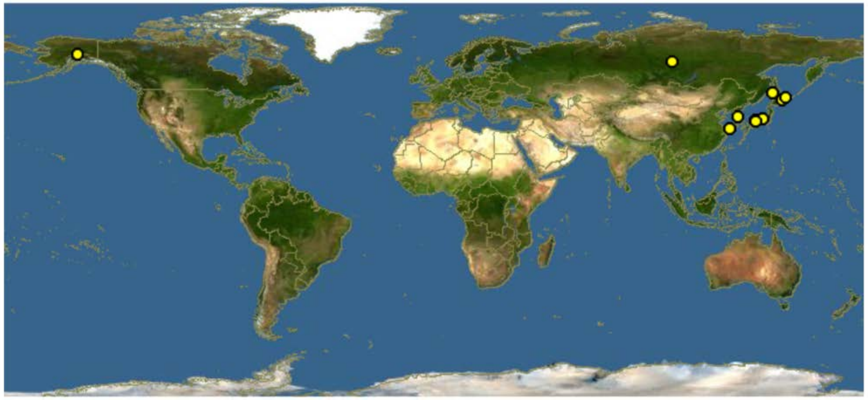
2017.09.22

Ecology of Sandfish



Scientific name :
Arctoscopus japonicus **(Steindachner**)
Spawning season : November to December
Lifespan : 6 year

Habitat of sandfish in the world

Ecology of Sandfish

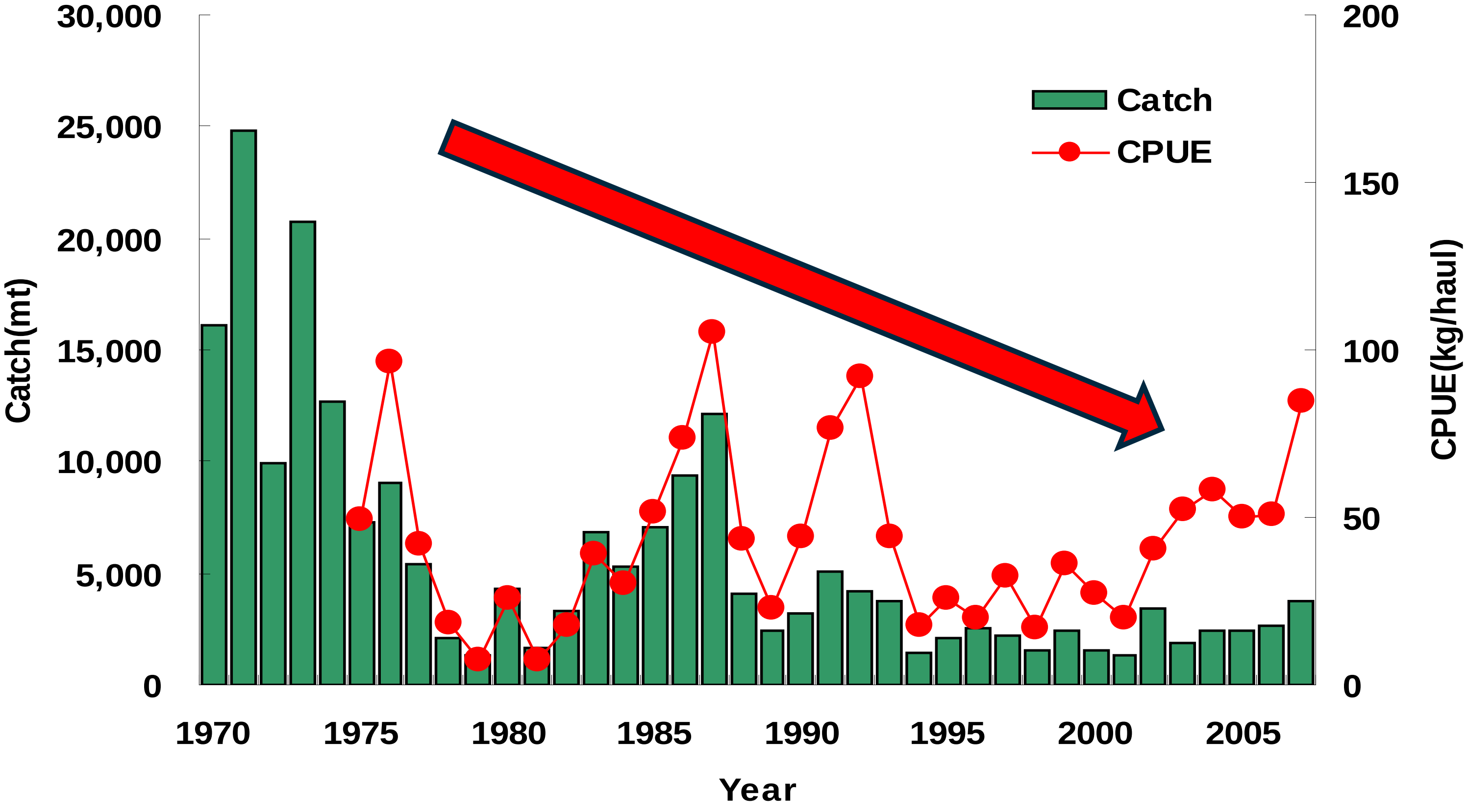


Figure. Catch and CPUE of sandfish from 1970 to 2009 in eastern waters of Korea

Measure to increase sandfish catch

1. Increasing survival rate of eggs using gulfweed.

2. Protecting juvenile sandfish by increasing the mesh size of the net.

3. Implementing total allowable catch.

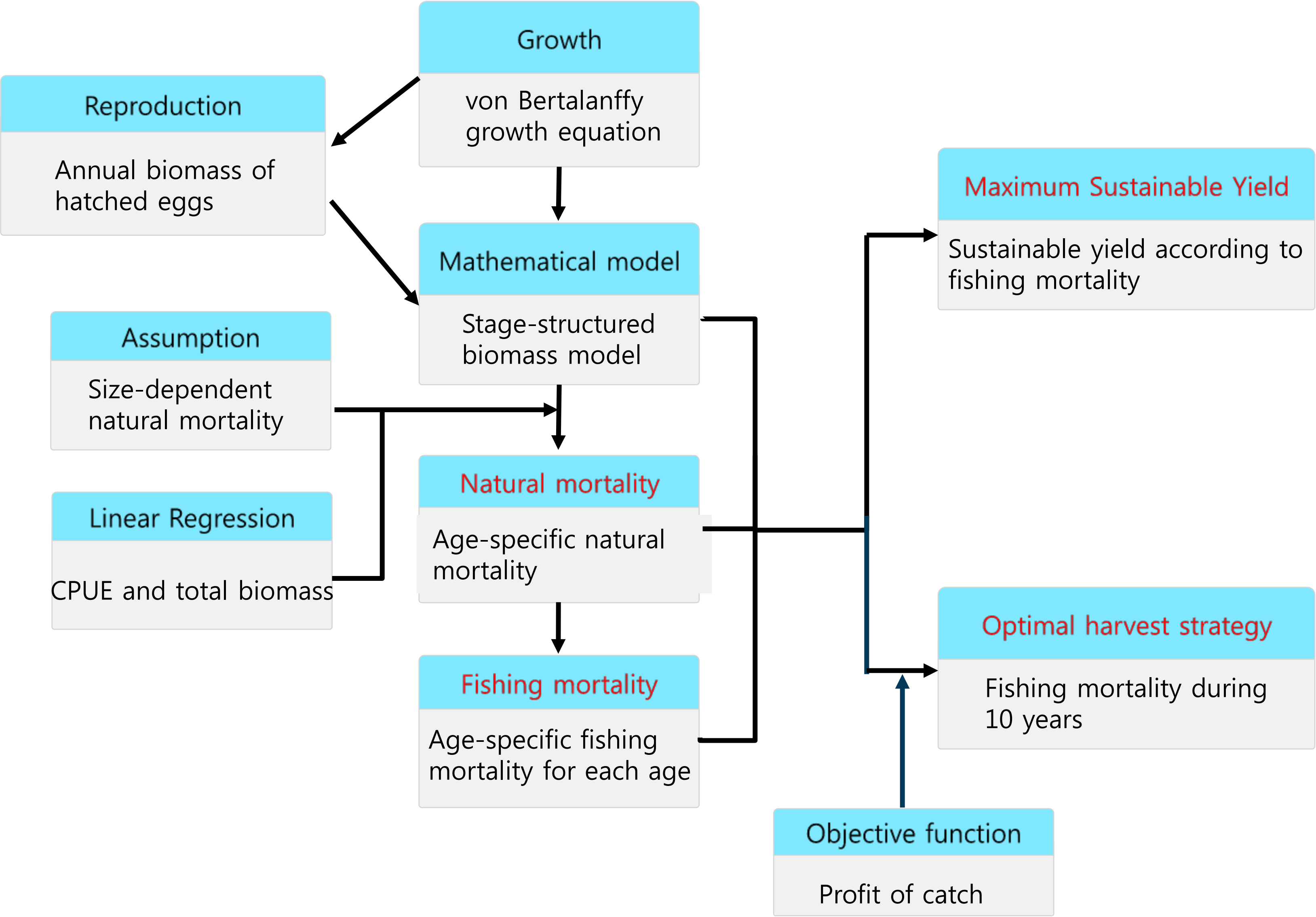
Question and Objective

Question : What is the best strategy to increase profit of catch while recovering the sandfish stock without cause fishermen's opposition?

Goal : To present an optimal harvest strategy for sustaining the maximum economic yield of sandfish in the East Sea in the long term.

Objective : To analyze methods by which we can attain the maximum sustainable yield and the maximum economic yield of sandfish by changing the monthly fishing effort.

Flow Chart for objective



Growth

von Bertalanffy growth equation

Reproduction

Annual biomass of hatched eggs

Mathematical model

Stage-structured biomass model

Assumption

Size-dependent natural mortality

Linear Regression

CPUE and total biomass

Natural mortality

Age-specific natural mortality

Fishing mortality

Age-specific fishing mortality for each age

Maximum Sustainable Yield

Sustainable yield according to fishing mortality

Optimal harvest strategy

Fishing mortality during 10 years

Objective function

Profit of catch

Assumption

1. Sandfish spawn only in December.

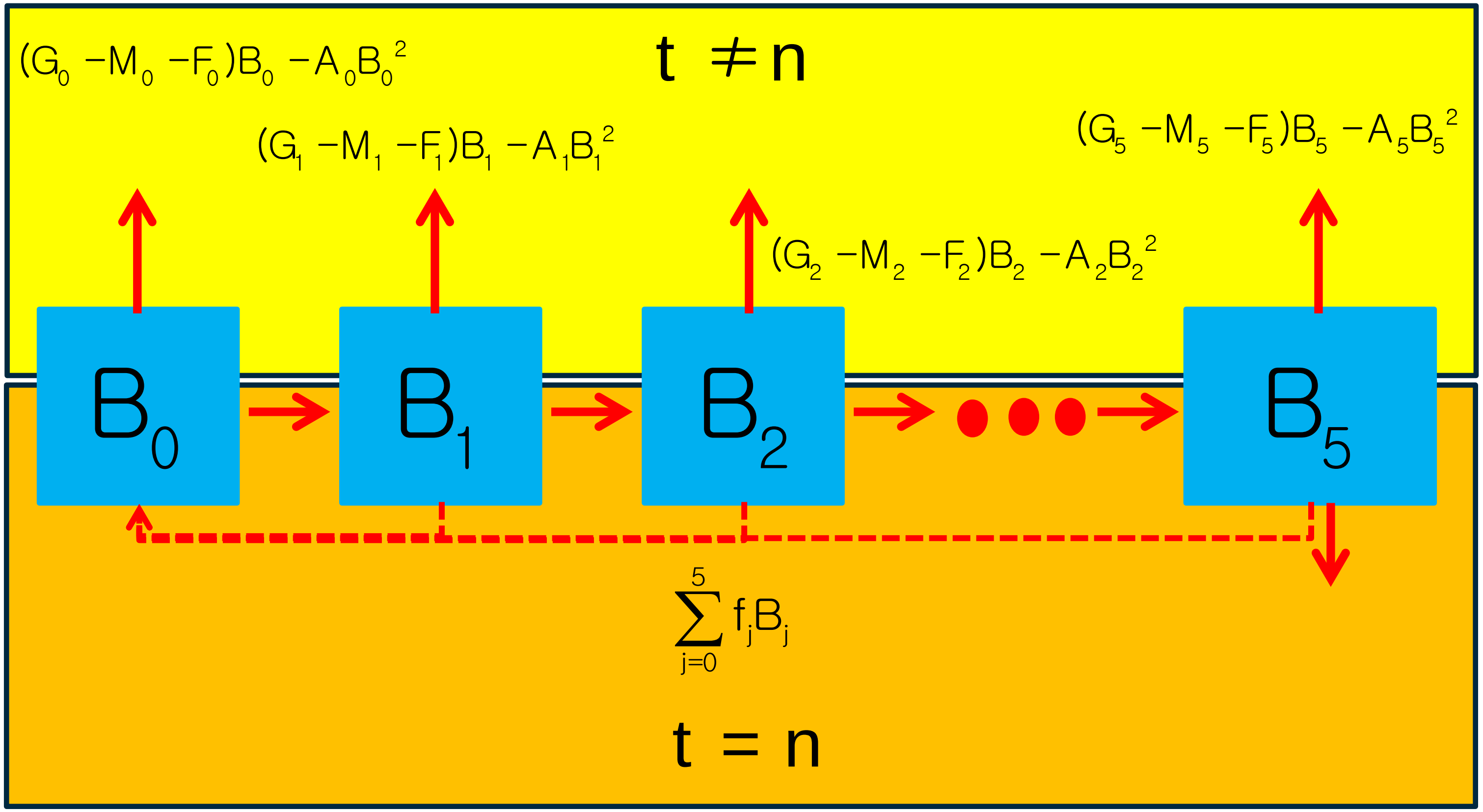
2. Sandfish hatch all eggs on the first day of January every year.

3. All stage of sandfish transfer to the next stage on the first day of January.

4. Because lifespan of sandfish is 6 years, all sandfish at stage 5 die in the next year.

Step 2 : Construct a stage-structured model

(Biomass increase) = (Growth – Natural mortality – Fishing mortality) x (Biomass) - (Density dependent mortality) x (Biomass)



$t \neq n$

$(G_0 - M_0 - F_0)B_0 - A_0 B_0^2$

$(G_1 - M_1 - F_1)B_1 - A_1 B_1^2$

$(G_2 - M_2 - F_2)B_2 - A_2 B_2^2$

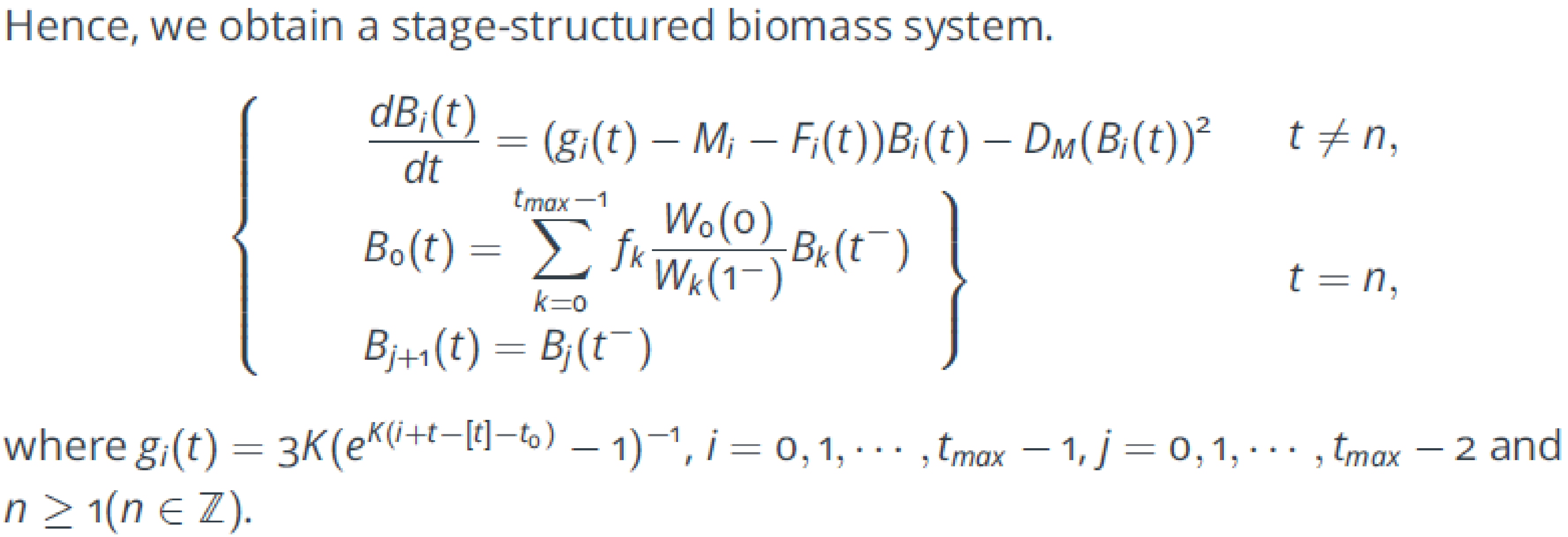
$(G_5 - M_5 - F_5)B_5 - A_5 B_5^2$

B_0 → B_1 → B_2 → ••• → B_5

$\sum_{j=0}^{5} f_j B_j$

$t = n$

Step 1 : Construct a dynamics model



Hence, we obtain a stage-structured biomass system.

$$\begin{cases} \dfrac{dB_i(t)}{dt} = (g_i(t) - M_i - F_i(t))B_i(t) - D_M(B_i(t))^2 & t \neq n, \\ \left.\begin{aligned} B_0(t) &= \sum_{k=0}^{t_{max}-1} f_k \frac{W_0(0)}{W_k(1^-)} B_k(t^-) \\ B_{j+1}(t) &= B_j(t^-) \end{aligned}\right\} & t = n, \end{cases}$$

where $g_i(t) = 3K(e^{K(i+t-[t]-t_0)} - 1)^{-1}$, $i = 0, 1, \cdots, t_{max} - 1$, $j = 0, 1, \cdots, t_{max} - 2$ and $n \geq 1 (n \in \mathbb{Z})$.

Size-dependent natural mortality

$$M(t) = \frac{q}{L(t)}$$

$$M_k = \int_k^{k+1} M(t)dt = \int_k^{k+1} \frac{q}{L(t)}dt = q\frac{\ln(e^{K(k+1-t_0)} - 1) - \ln(e^{K(k-t_0)} - 1)}{KL_\infty}.$$

$$M_0 = \frac{q}{KL_\infty} \ln\left(\frac{e^{K(1-t_0)} - 1}{e^{-Kt_0} - 1}\right),$$

$$M_i = V_i M_0,$$

where $$V_i = \frac{\ln((e^{K(i+1-t_0)} - 1)/(e^{K(i-t_0)} - 1))}{\ln((e^{K(1-t_0)} - 1)/(e^{-Kt_0} - 1))}.$$

Method of the estimation

For $n = 0,\ 1,\ 2, \cdots$

Assuming $B(0) = \begin{bmatrix} \vdots \\ B_i(0) \\ \vdots \end{bmatrix}$

$$\Rightarrow \quad \mathrm{CPUE}_n = \alpha \int_n^{n+1} \sum_{i=1}^{5} B_i(t)\,dt$$

Least square parameter estimation

Initial biomass	Natural Mortality	Catchability coefficient	Error
$B_{0,1}$	$M_{0,1}$	α_1	5000
$B_{0,2}$	$M_{0,2}$	α_2	4000
$B_{0,3}$	$M_{0,3}$	α_3	3000
$B_{0,4}$	$M_{0,4}$	α_4	2000
$B_{0,5}$	$M_{0,5}$	α_5	3000
$B_{0,6}$	$M_{0,6}$	α_6	4000
⋮	⋮	⋮	⋮

Parameter estimation : Maturity

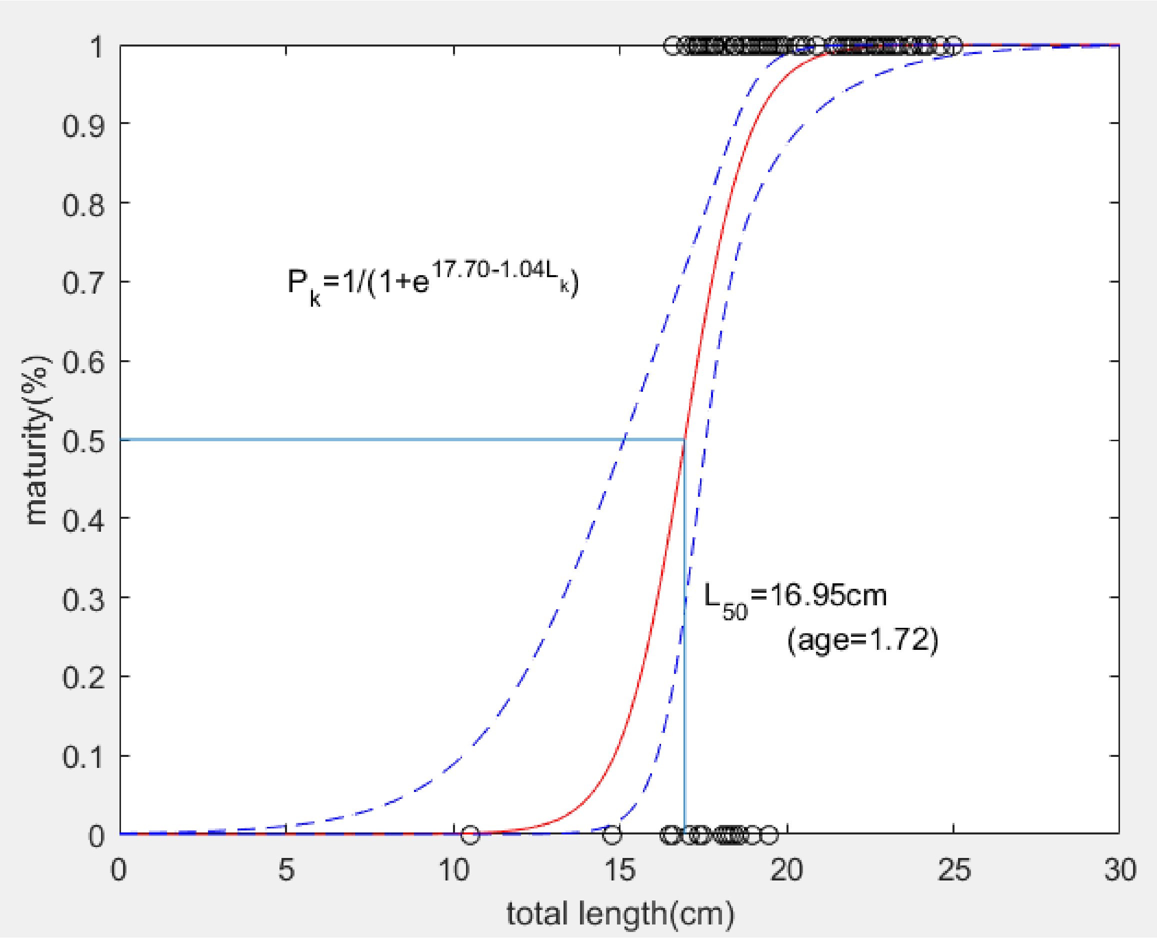


Figure. A logistic regression curve for probability of participation for spawning about total length using 126 female sandfish during December 2005 and December 2006 in the East Sea.

Parameter estimation : Growth Equation

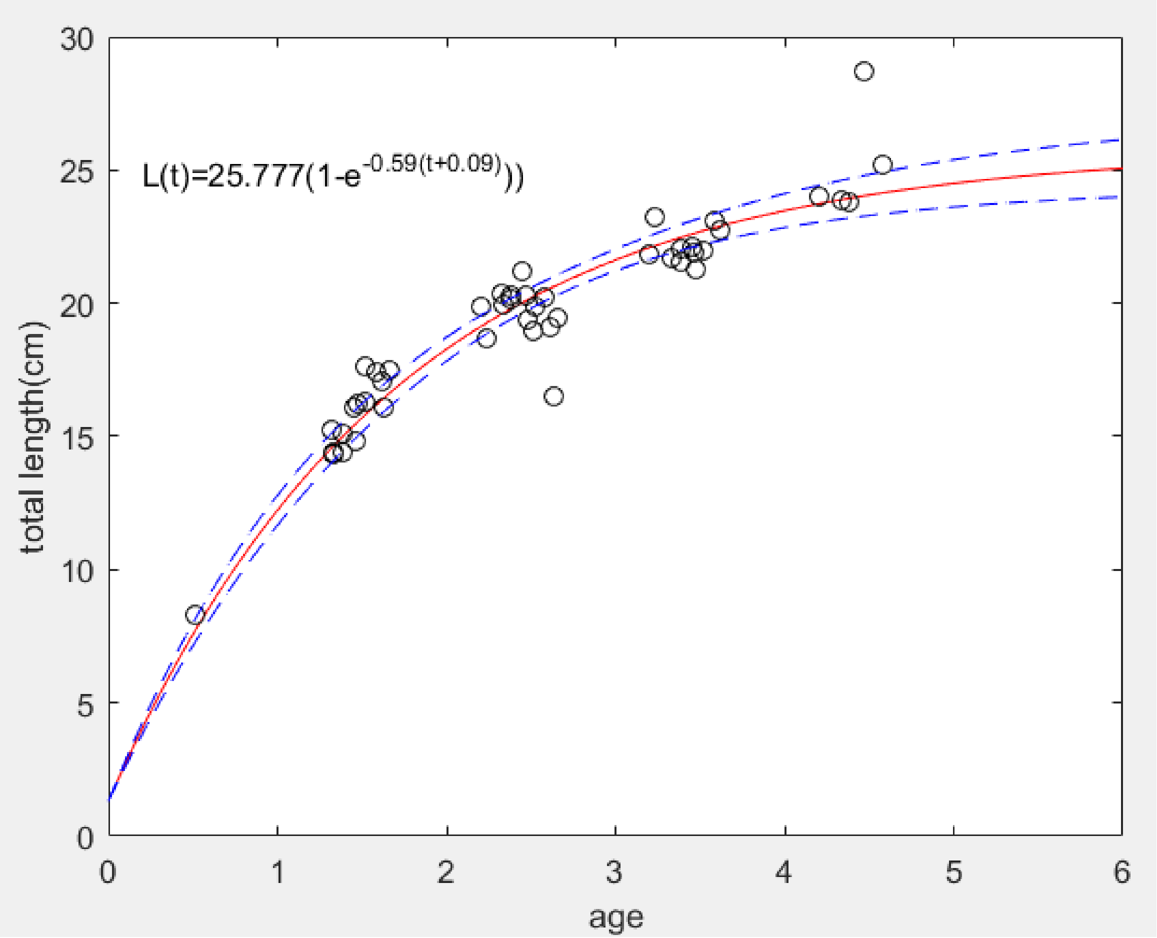


Figure. A nonlinear regression curve for von Bertalanffy growth equation based on annulus counting of otoliths extracted from 46 female sandfish collected from 2005 to 2008 in the East Sea

Parameter estimation : Natural mortality

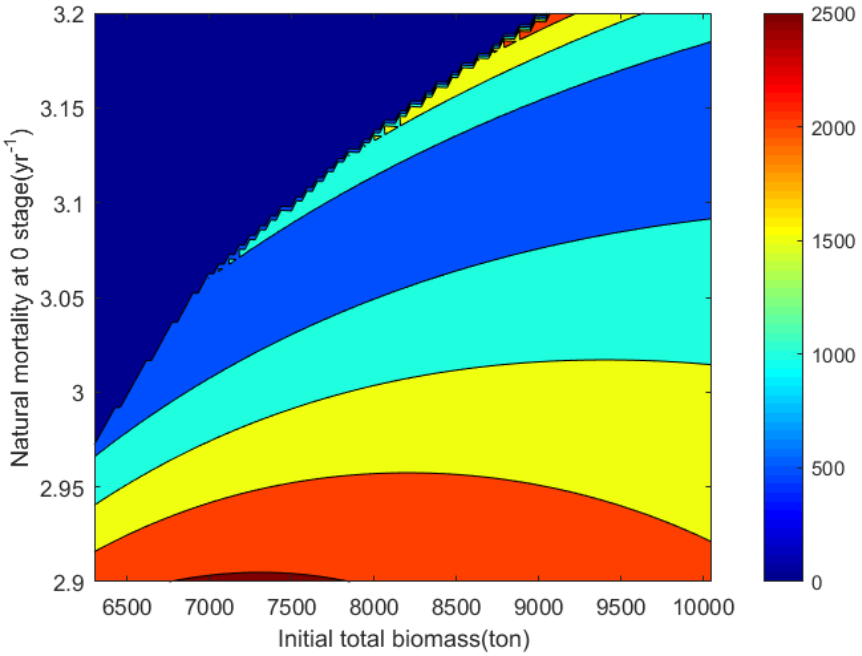
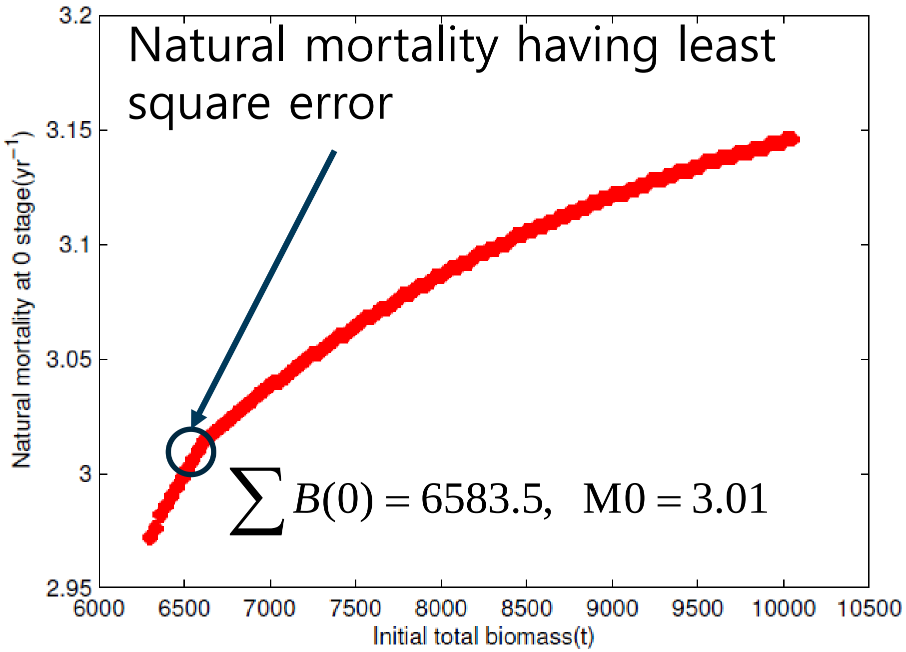


Figure. Variation of least square errors for initial total biomass and natural mortality at stage 0


Natural mortality having least square error

$$\sum B(0) = 6583.5, \quad \mathrm{M0} = 3.01$$

Figure. Natural mortality at stage 0 corresponding to least square errors for each initial total biomass

Parameter estimation : Fishing mortality

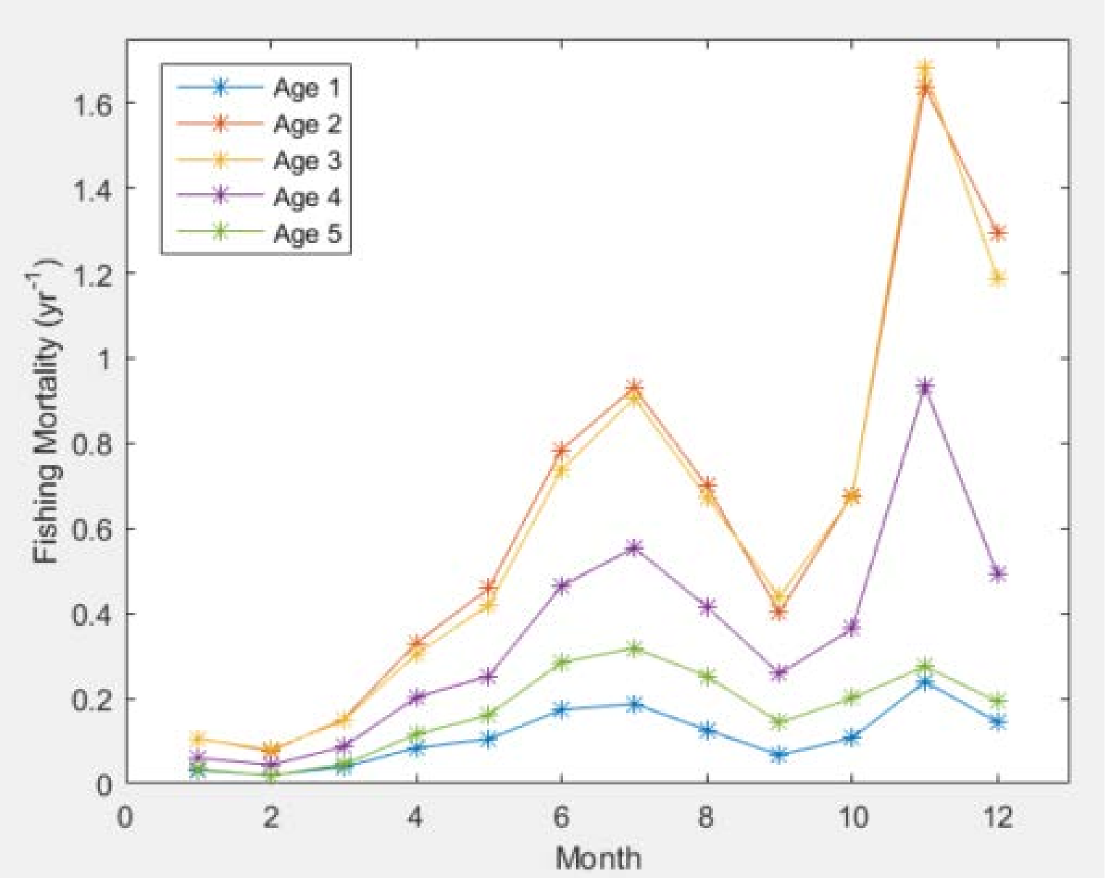


Figure. Monthly-averaged fishing mortality of sandfish from 2000 to 2009 in eastern Costal waters of Korea when $\sum B(0) = 6583.5, \ \ \mathrm{M0} = 3.01$

Maximum Sustainable Yield

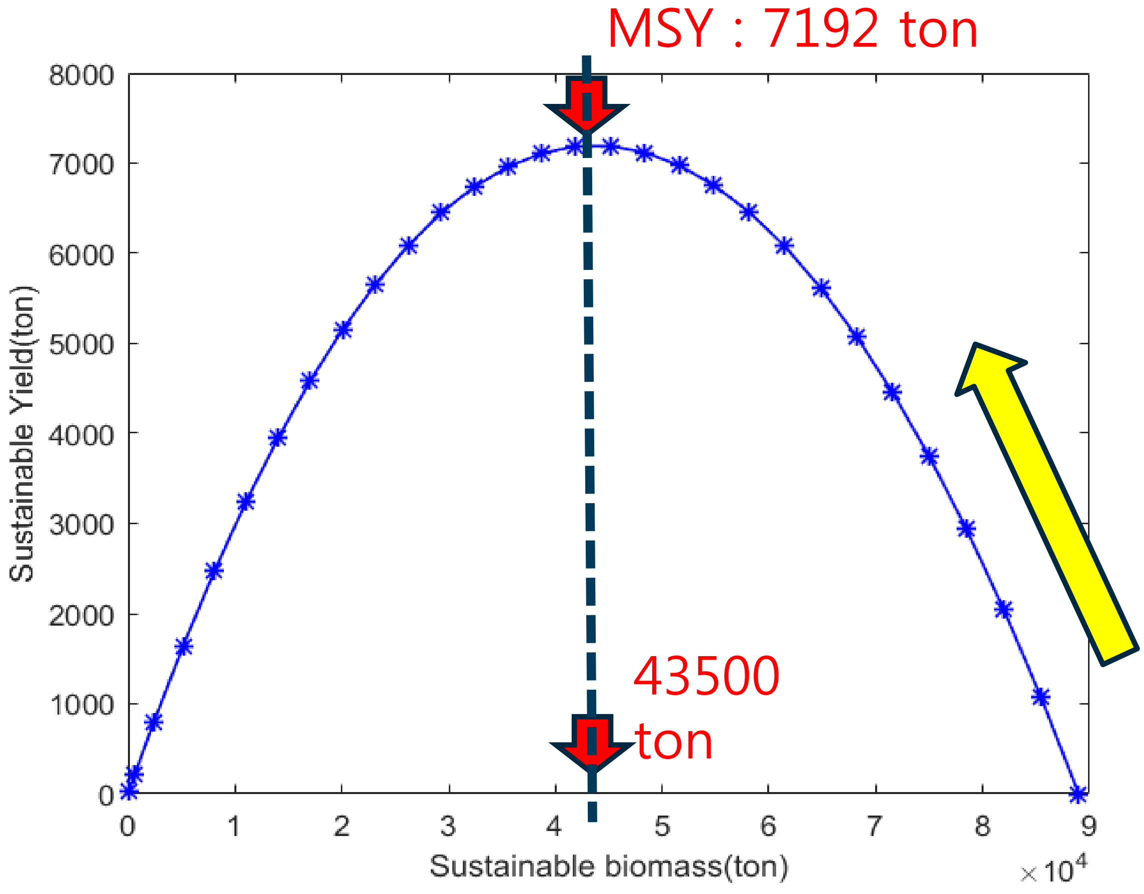


Figure. Pair of sustainable biomass and sustainable yield of sandfish according to fishing effort

Monthly price of sandfish

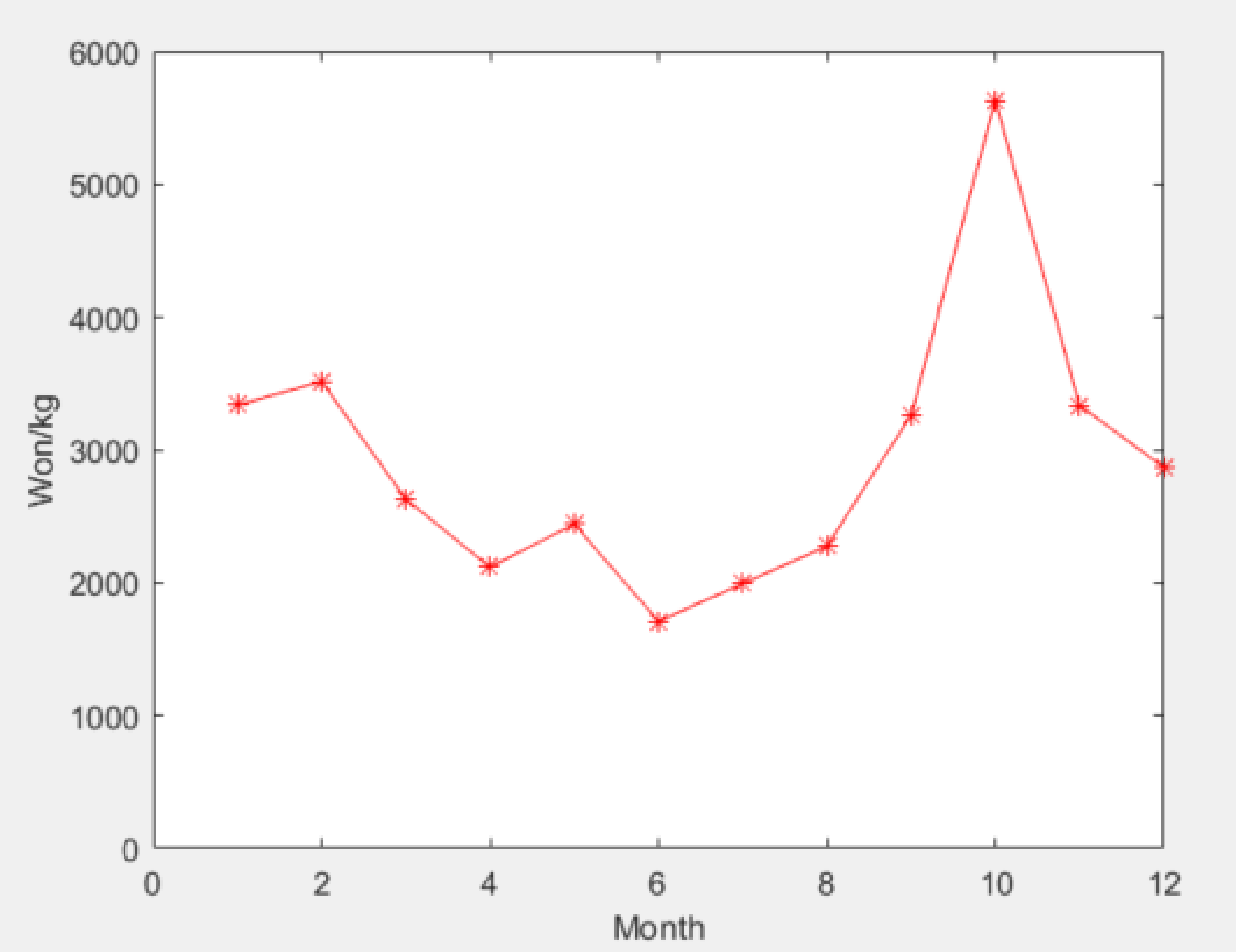


Figure. Monthly average price of sandfish in last 10 years

Maximum Economic Yield

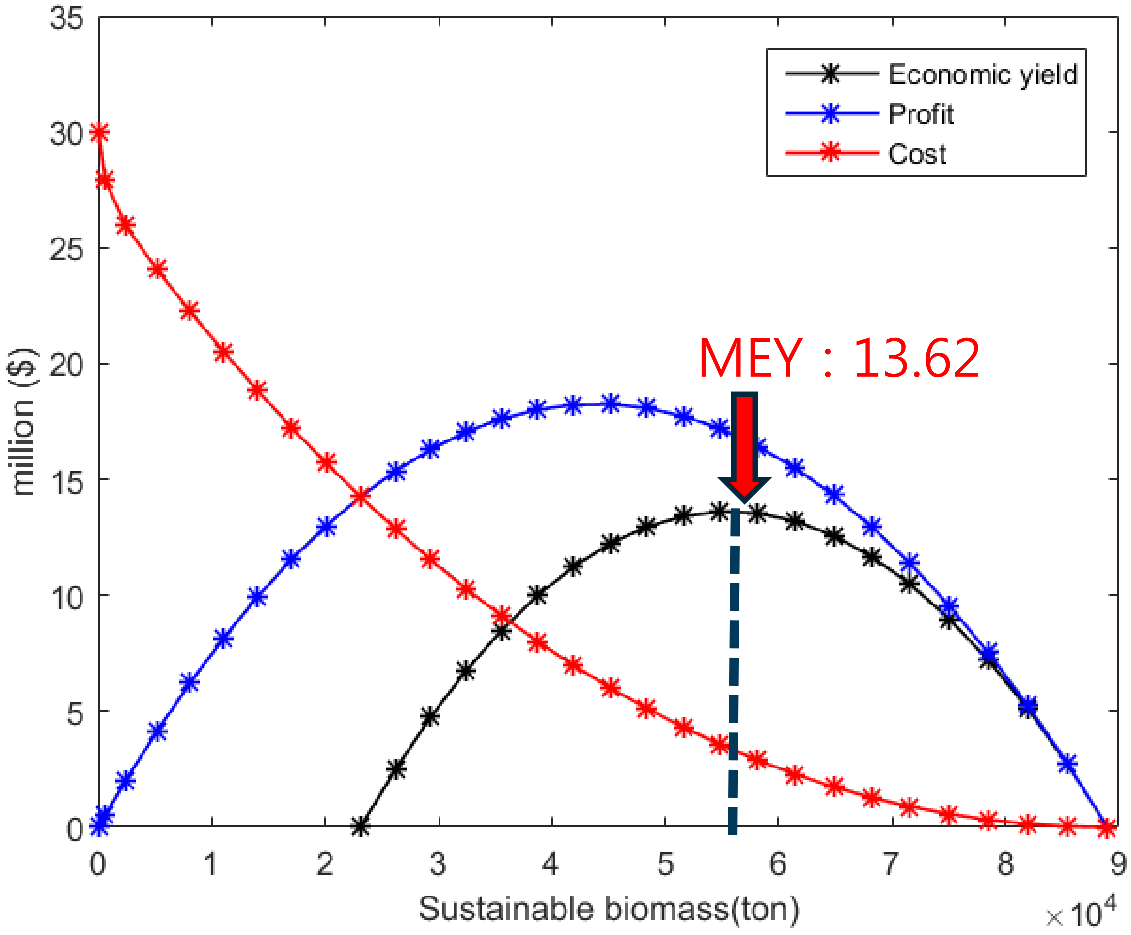


Figure. Pairs of sustainable biomass & economic yield, profit and cost of sandfish according to fishing effort

Optimal Harvest Problem

Objective functional

$$\boxed{\max J(u) = \sum_{n=0}^{N} \int_{n}^{(n+1)^-} [\overbrace{C_1 \sum_{i=0}^{5} \alpha_i u_n(t) B_{i,n}(t)}^{\text{Profit for total catch}} - \overbrace{C_2 u_n(t)^2}^{\text{Cost for catch}}]\,dt + \underbrace{\sum_{i=0}^{5} B_{i,N}((N+1)^-)}_{\text{Maintenance factor for the biomass}}}$$

subject to

$$\begin{cases} \dfrac{dB_i(t)}{dt} = (g_i(t) - M_i - \alpha_i u(t))B_i(t) - D_M(B_i(t))^2 & \text{if } t \neq n, \\ \left. \begin{aligned} B_0(t^+) &= \sum_{i=2}^{5} f_i \frac{W(0)}{W((i+1)^-)} B_i(t^-) \\ B_{i+1}(t^+) &= B_i(t^-) \end{aligned} \right\} & \text{if } t = n, \end{cases}$$

where $g_i(t) = 3K(e^{K(p+t-[t]-t_0)} - 1)^{-1}$ for $i = 0, 1, \cdots, 5$ and $n \geq 1 (n \in \mathbb{N})$. The control variable $u(t)$ represents fishing effort at time t.

$$m_1 \leq u_n(t) \leq m_2, \text{ and} t \in [n, n+1], \text{ for } n = 0, \cdots, N$$

Results: Optimal harvest strategy

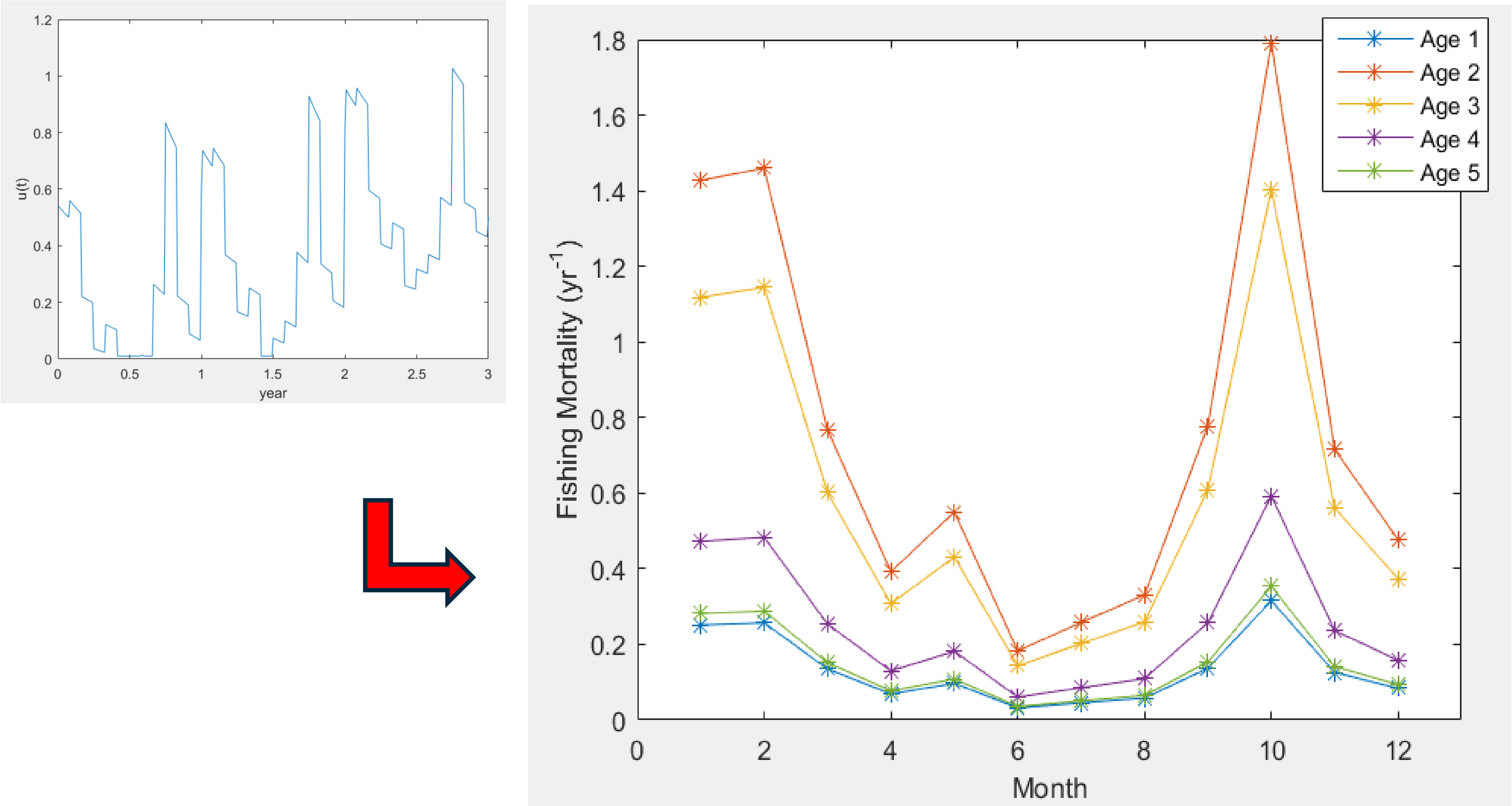
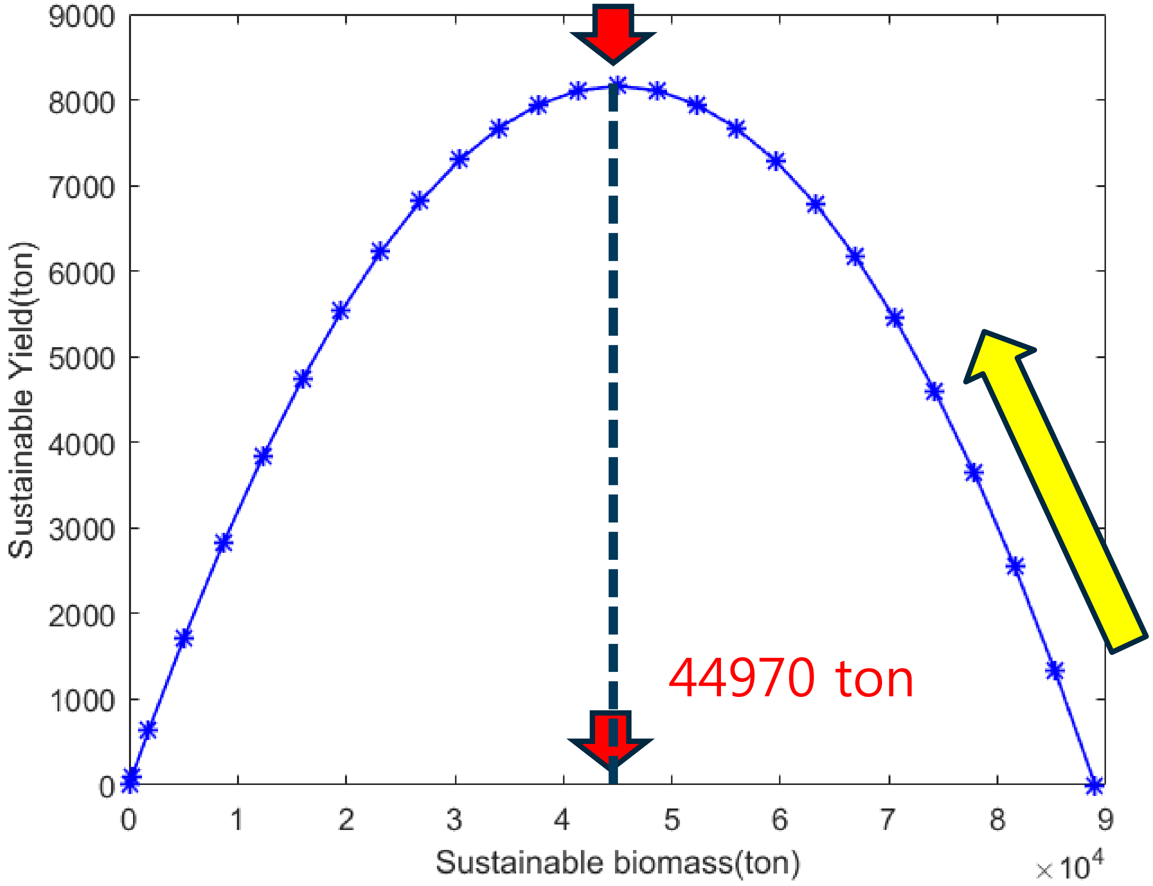


Figure. Monthly fishing mortality for each age applied optimal harvest strategy using average price for the last 10 years.

Results: Maximum Sustainable Yield

MSY : 8163 ton



44970 ton

Figure. Pair of sustainable biomass and sustainable yield considering optimal harvest strategy of sandfish according to fishing effort

Results: Maximum Sustainable Yield

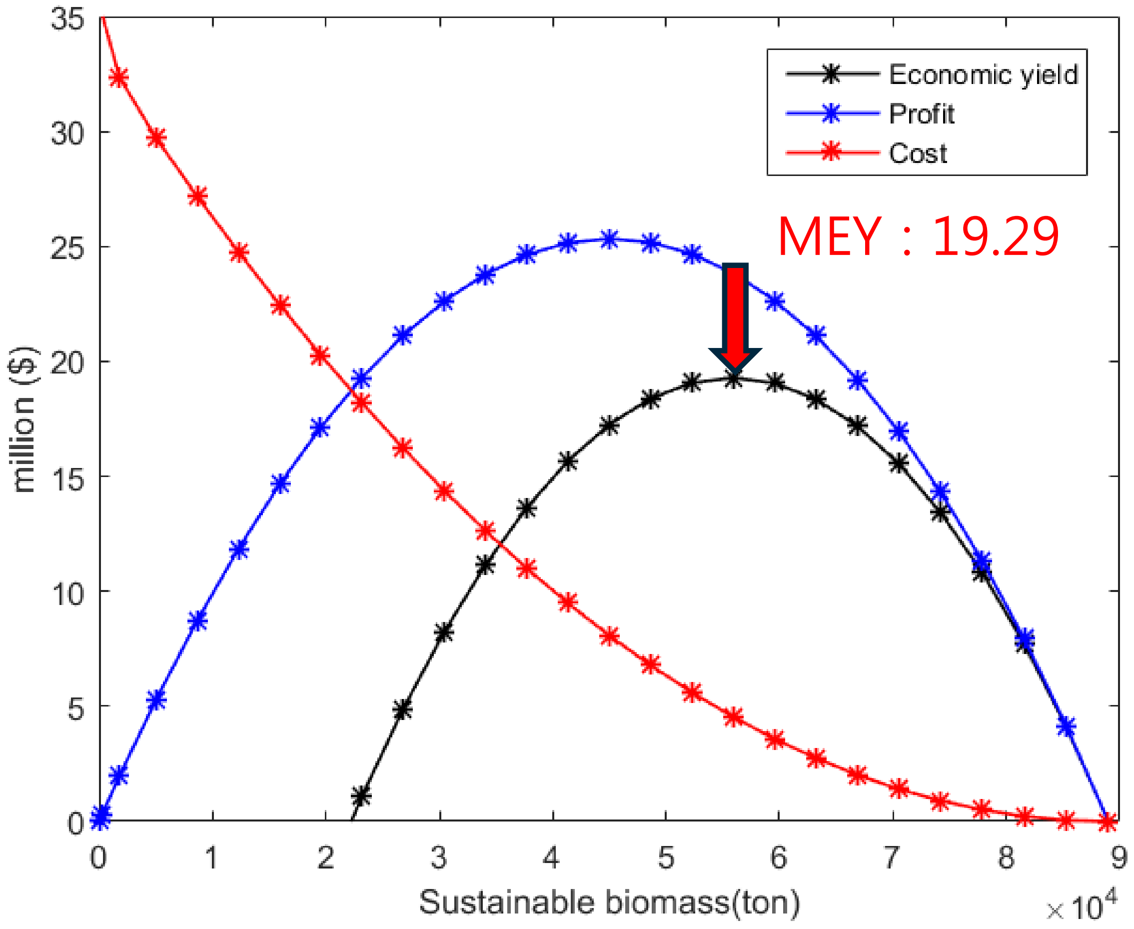
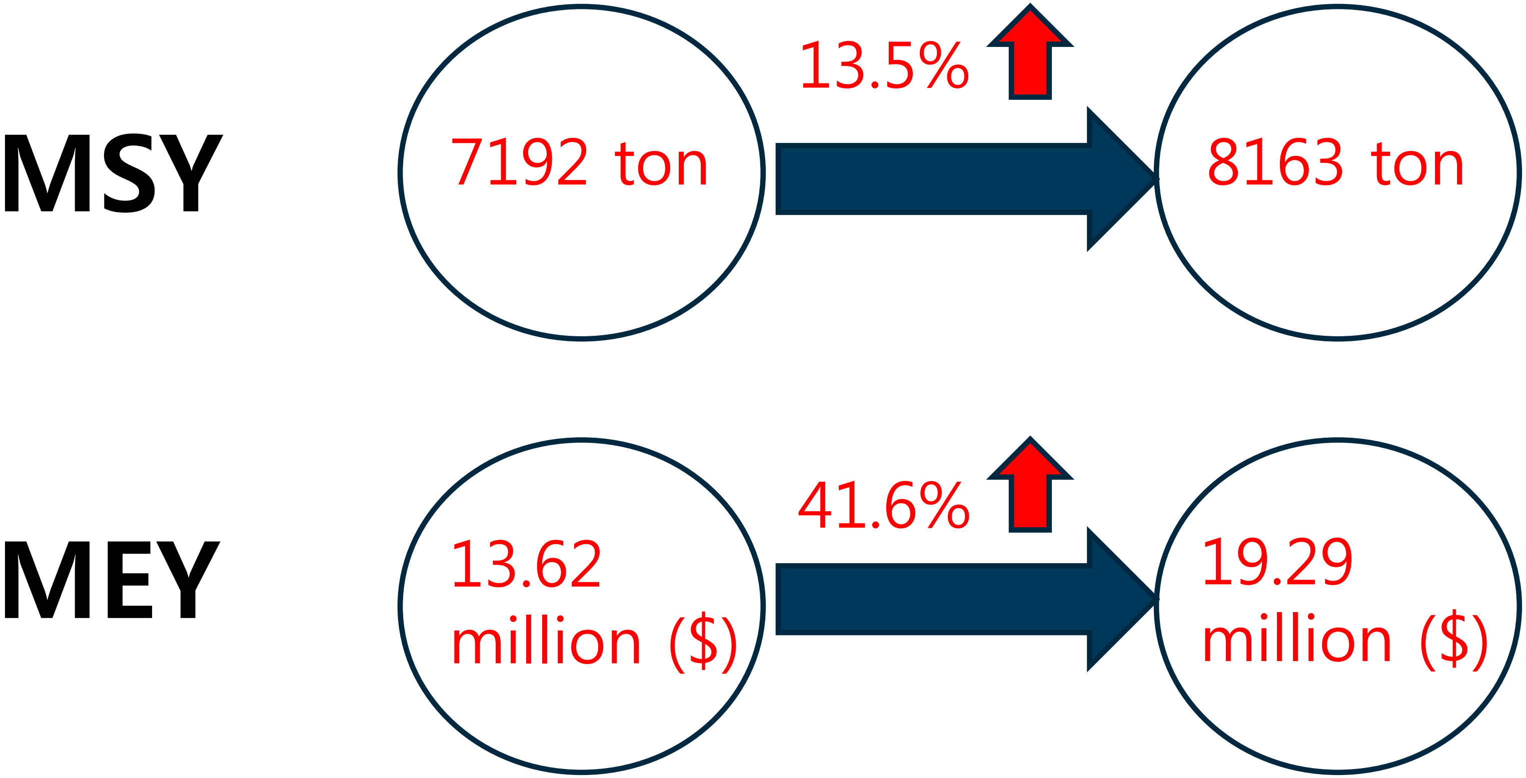


Figure. Pairs of sustainable biomass & economic yield, profit and cost considering optimal harvest strategy of sandfish according to fishing effort

Conclusion



MSY

7192 ton

13.5%

8163 ton

MEY

13.62 million ($)

41.6%

19.29 million ($)

Summary

1. We constructed a stage-structured biomass model to propose the optimal harvest strategy.

2. We can maximize MEY by catching just before and after spawning when considering monthly variable market price of sandfish.

3. We expect that MSY and MEY can be increased by implementing 13.5% and 41.6% due to the changing monthly fishing effort.

Thank you for your attention.