Scaling of the Mixed Layer Depth under Surface Heating by Using LES

Y. Choi and Y. Noh

Department of Atmospheric Sciences

Yonsei University, Korea

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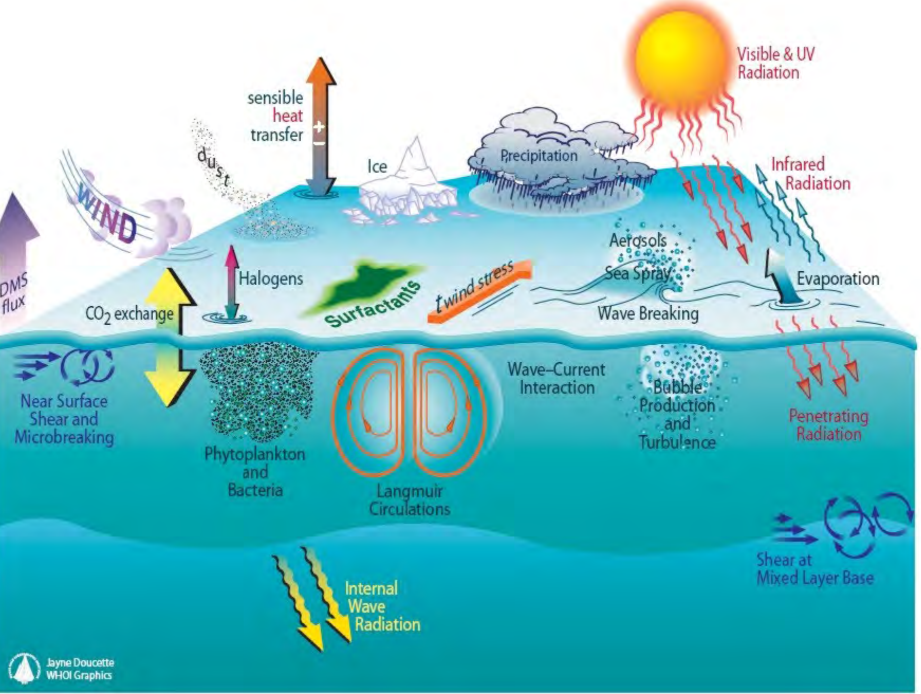
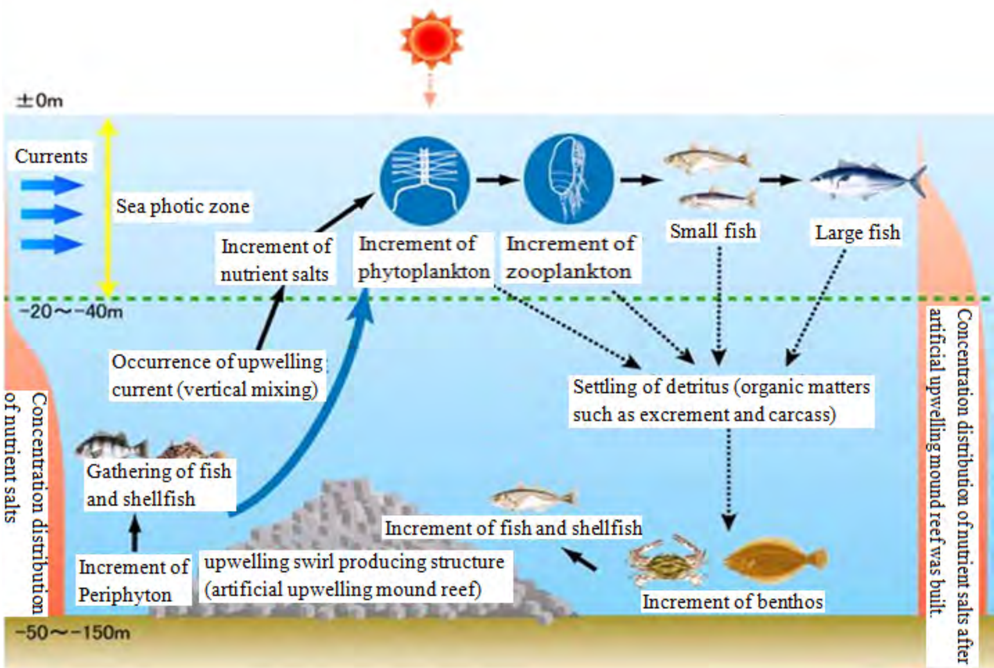
Comments on “Langmuir Turbulence and Surface Heating in the Ocean Surface Boundary Layer.”
J. Phys. Oceanogr.

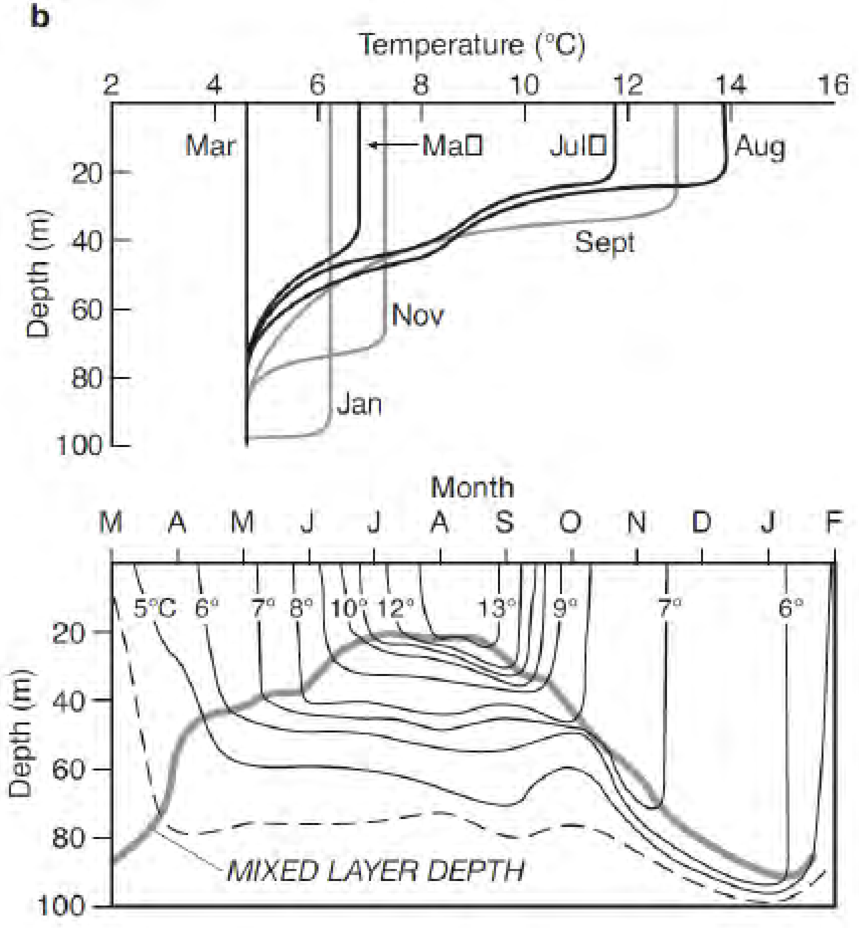
Ocean Mixed Layer

- strong turbulence due to convection or wind stress
- vertically uniform temperature
- important factor in vertical mixing
- determines downward transport of heat, and thus sea surface temperature
  ⟶ affects the climate
- determines how much deep, nutrient rich water will be brought
  to the surface to feed the phytoplankton





Seasonal Variation of the Ocean Mixed Layer



During the heating season,
a seasonal thermocline is formed.

Prediction of the Depth of a Thermocline

(Kraus and Turner 1967)

w_e : entrainment velocity
ΔB : buoyancy jump across MLD
Q_0 : surface buoyancy flux
u_* : frictional velocity
ε_m : mean dissipation within the mixed layer

1) Integration of TKE equation over MLD (h)

$$\Rightarrow \boxed{w_e}\Delta B = Q_0 + 2m_1 u_*^3 / h - \varepsilon_m$$

$\partial_t h$ source/sink terms of TKE within the mixed layer

Surplus of TKE within the mixed layer is used to deepen MLD.

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2) total dissipation over h ∝ source terms

$$h\varepsilon_m = m_d u_*^3 + 0.25(1-n)h[|Q_0| - Q_0]$$

$$\Rightarrow w_e \Delta B = 0.5h[(1-n)|Q_0| + (1+n)Q_0] + 2mu_*^3$$

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3) w_e (= $\partial h/\partial t$) = 0 during the formation of a thermocline ($Q_0 > 0$),

⇨ $$h = 2mL_{MO}$$ $L_{MO} = u_*^3/Q_0$: Monin-Obukhov length scale

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$$\Rightarrow \boxed{h = 2mL_{MO}}$$

$L_{MO} = u_*^3/Q_0$: Monin-Obukhov length scale

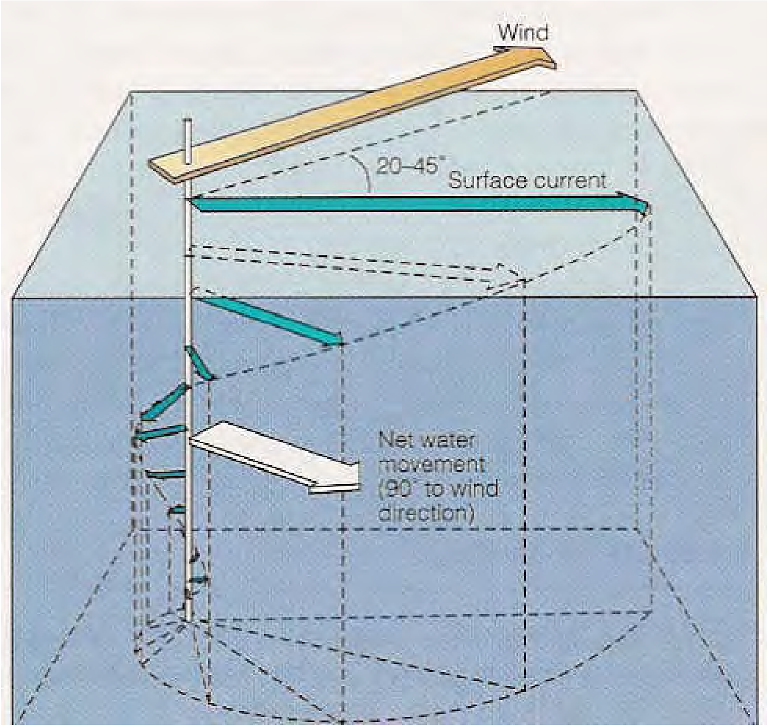
↔ *balance between the generation of turbulence by **wind stress***
*vs. the suppression of turbulence by **surface heating***

The downward transport of momentum is limited to the Ekman length scale.

⇨ $\lambda = u_* / f$

Ekman length scale

Coriolis parameter



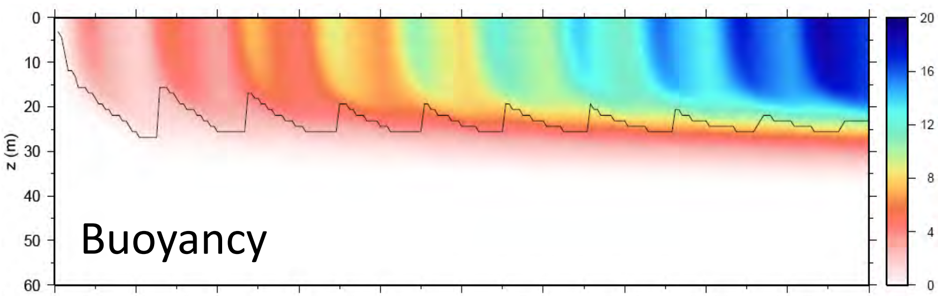
⇨ ***Is the depth of a thermocline affected by the Coriolis force?***

Investigation of the Formation of a Seasonal Thermocline Using LES

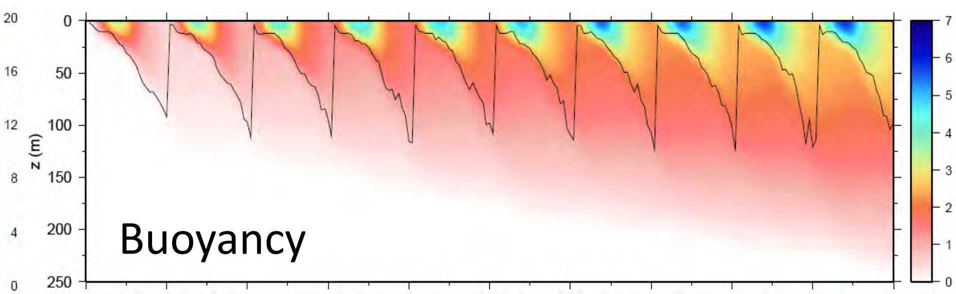
(Goh and Noh, *OD* 2013)

- The Coriolis force is found to play a fundamental role.

$\emptyset = 40°N$



$\emptyset = 0°$

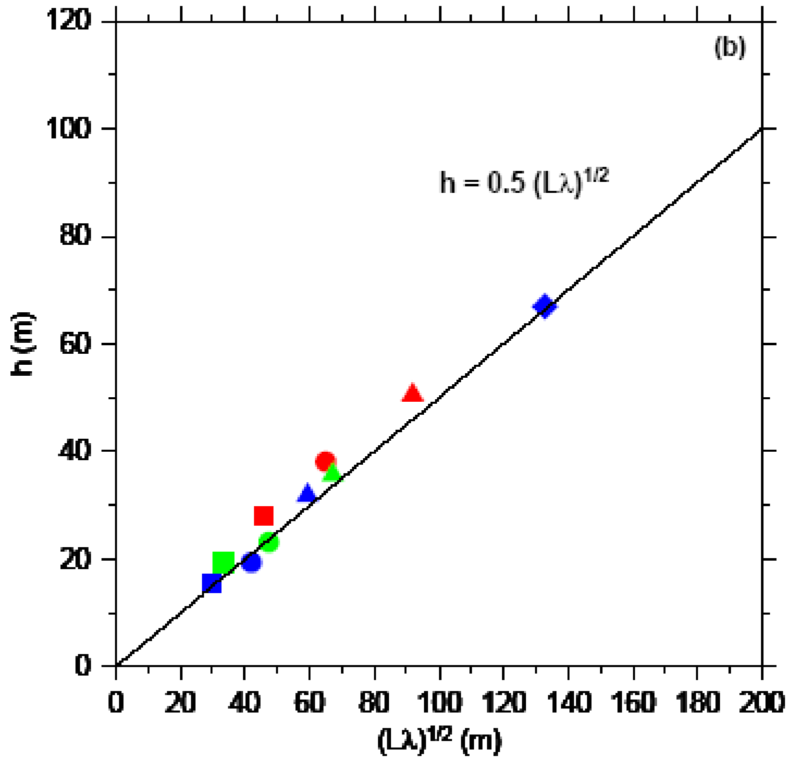


- 40°N - A thermocline is formed at a certain depth.
  No downward heat transport across the thermocline.

- Eq. - Heat continues to propagate downward to the deeper ocean.
  A well-defined thermocline is not formed.

Depth of a Seasonal Thermocline

(Goh and Noh, *OD* 2013)



$$h \propto (L_{MO}\lambda)^{1/2} \propto u_*^2 / (Q_0 f)^{1/2}$$

* $\lambda = u_* / f$ Ekman length scale

$L_{MO} = u_*^3 / Q_0$ Monin-Obukhov length scale

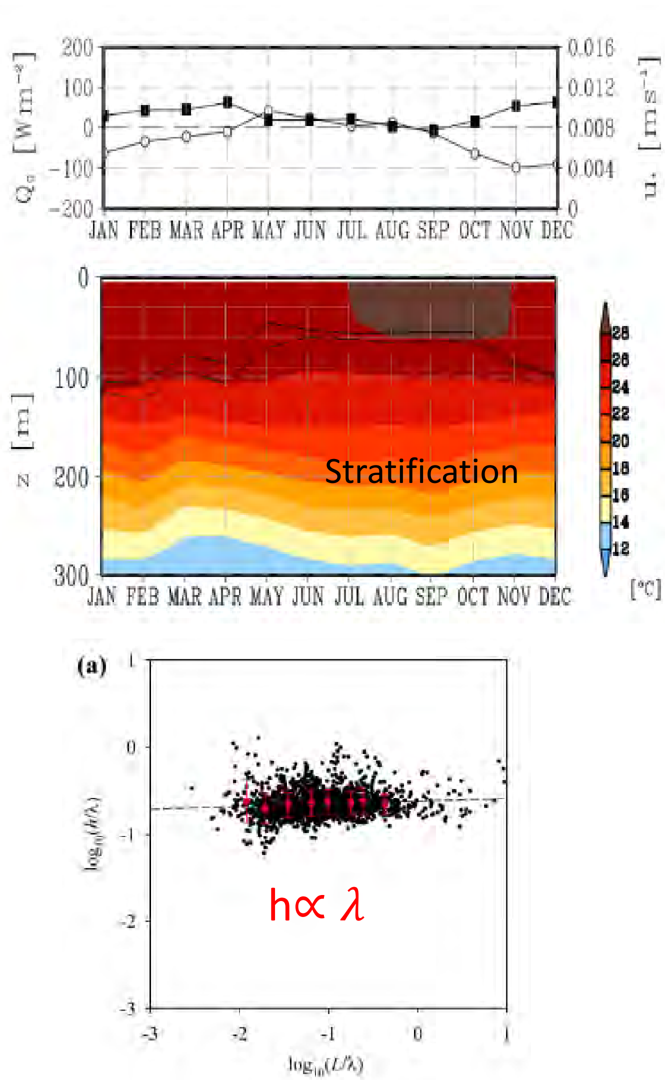
L_Z (= $(L_{MO}\lambda)^{1/2}$) : Zilitinkevich scale

They suggested the scale of the depth of a seasonal thermocline as Zilitinkevich scale.

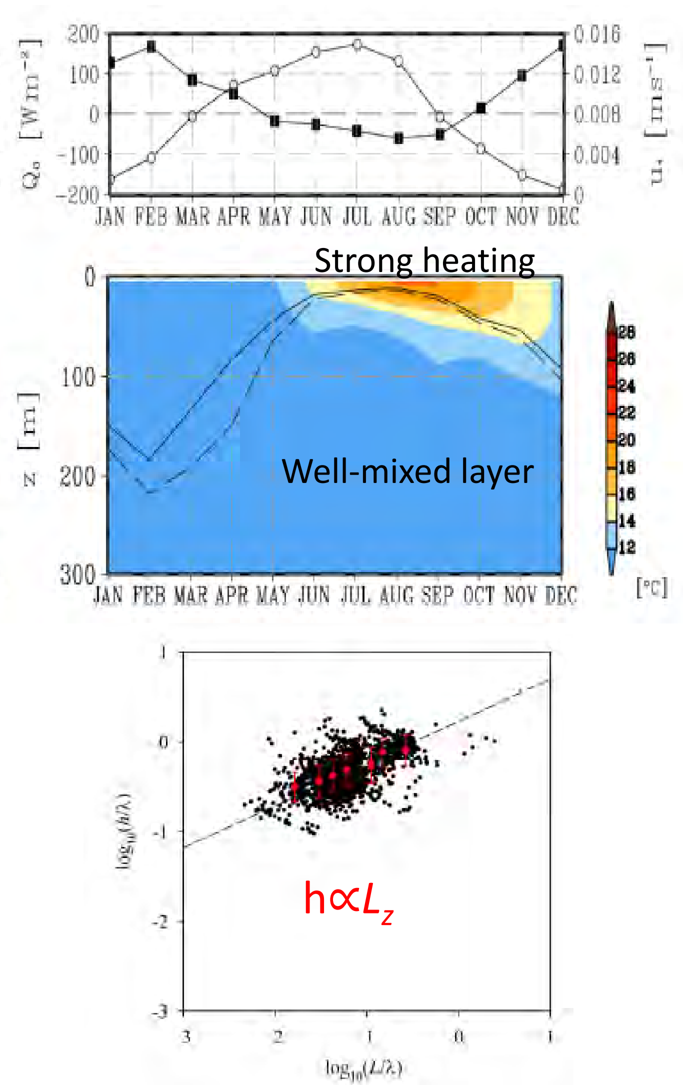
Response of the Upper Ocean to Surface Heating in the N. Pacific

(Lee et al. *JGR* 2015)

(15N, 180E)

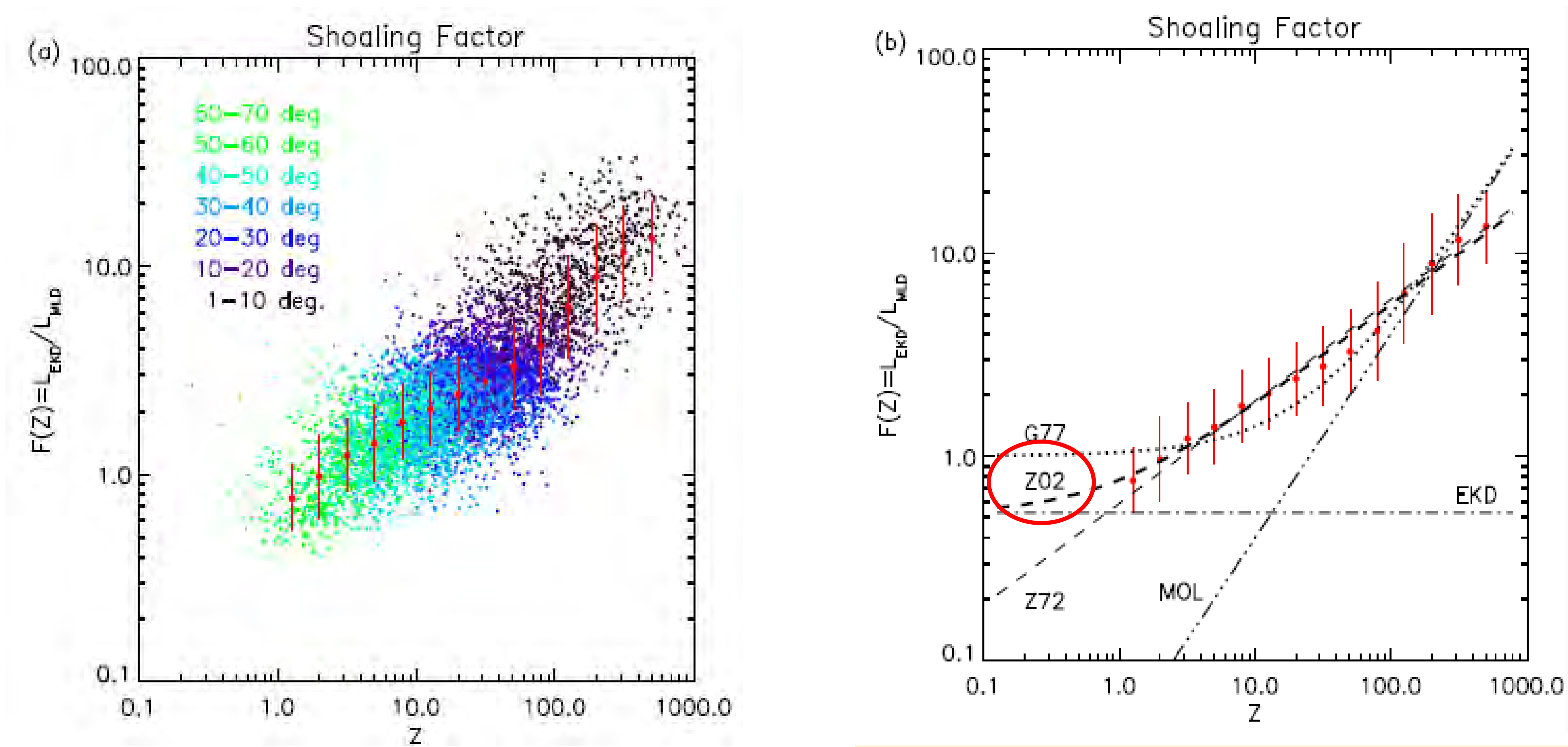


(40N, 180E)



When a seasonal thermocline is formed from the homogeneous layer, $h \propto L_z$

Scaling Surface Mixing/Mixed Layer Depth under Stabilizing Buoyancy Flux (Yoshikawa, *JPO* 2015)



L_{EKD} = Ekman length scale

Z02 = Zilitinkevich length scale

MOL = Monin-Obukhov length scale

- L_z is more suited for observed mixed layer depth than other length scales

Interpretation of the New Scale of *h* (L_z)

(Goh and Noh, *OD* 2013)

Kraus & Turner (1967)

$$w_e \Delta B = 0.5h[(1-n)\left|Q_0\right| + (1+n)Q_0] + 2mu_*^3$$

$w_e = 0$ during the formation of a seasonal thermocline ($Q_0 > 0$),

⇨ $h \propto L_{MO} (= u_*^3 / \mathrm{Q}_0)$

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***If the contribution of wind stress decreases with Ro (= $\lambda/h(=\frac{u_*}{fh})$)**

$$w_e \Delta B = 0.5h[(1-n)|Q_0| + (1+n)Q_0] + 2mu_*^3(\lambda / h)$$

λ : Ekman length

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⇨ $h \propto (L_{MO}\lambda)^{1/2}$ (= L_Z : Zilitinkevich scale)

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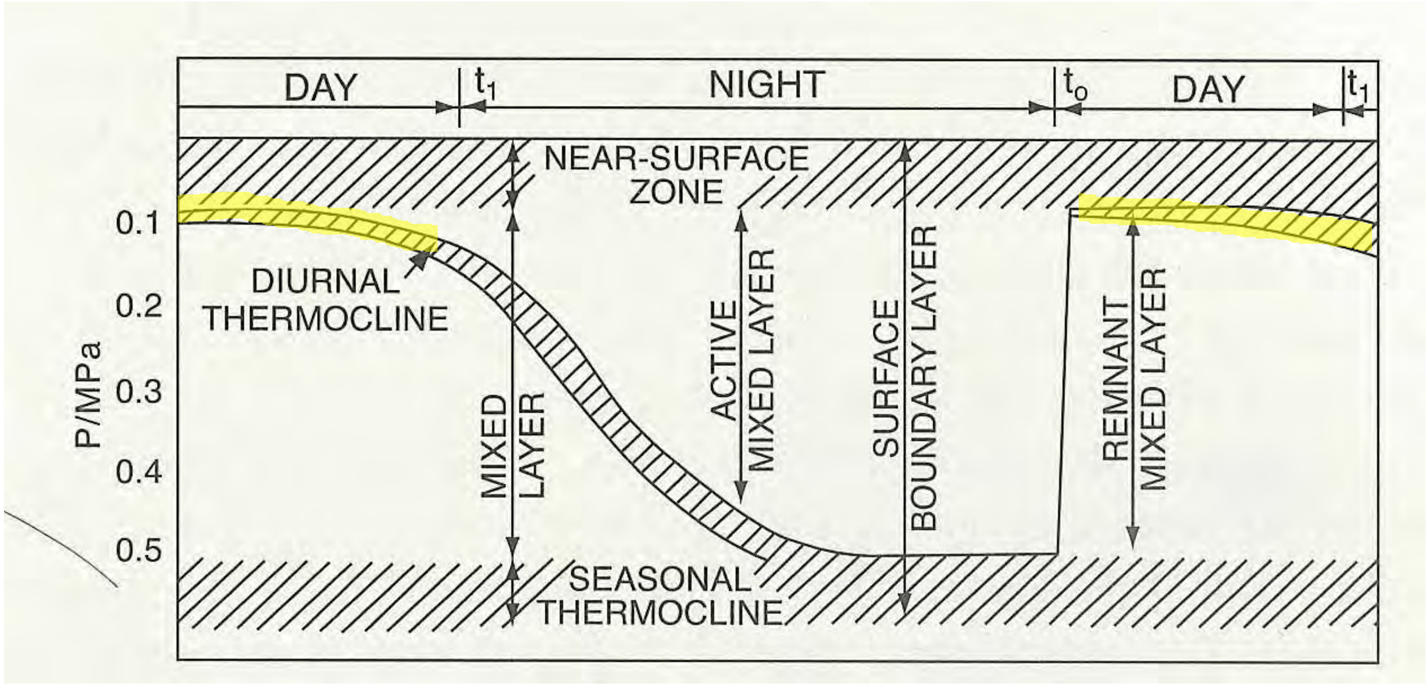
⇨ $h \propto (L_{MO}\lambda)^{1/2}$ (= L_Z : Zilitinkevich scale)

The scale for the seasonal thermocline should be L_z.

⟶ What about diurnal thermocline?

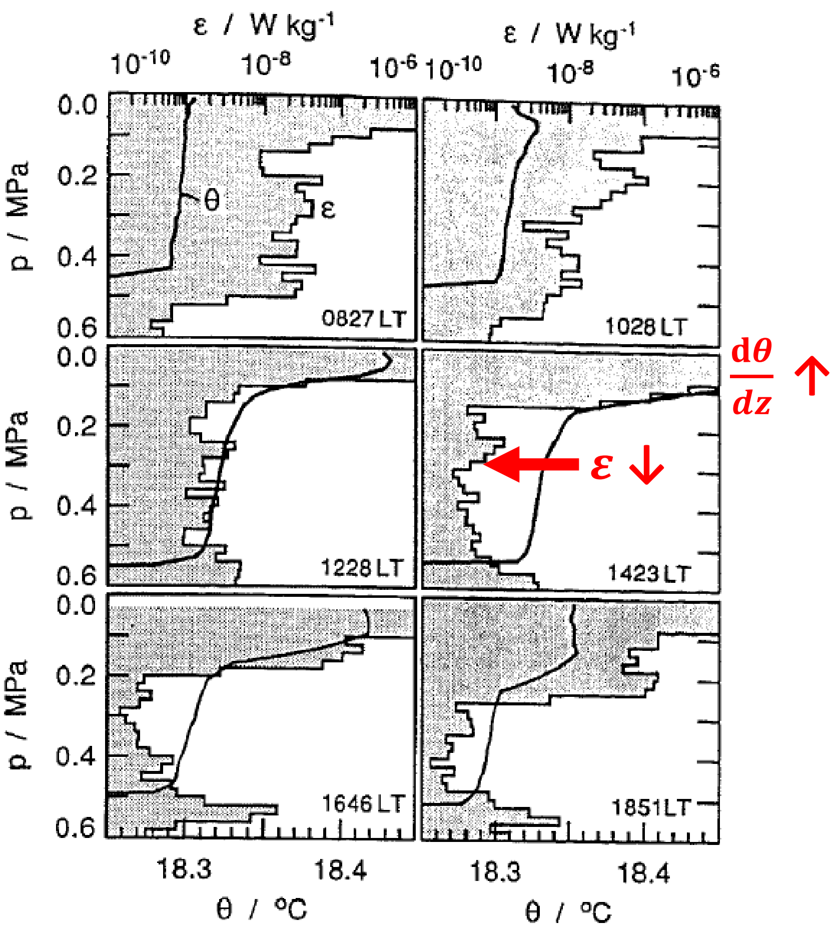
Diurnal Variation of the Ocean Mixed Layer



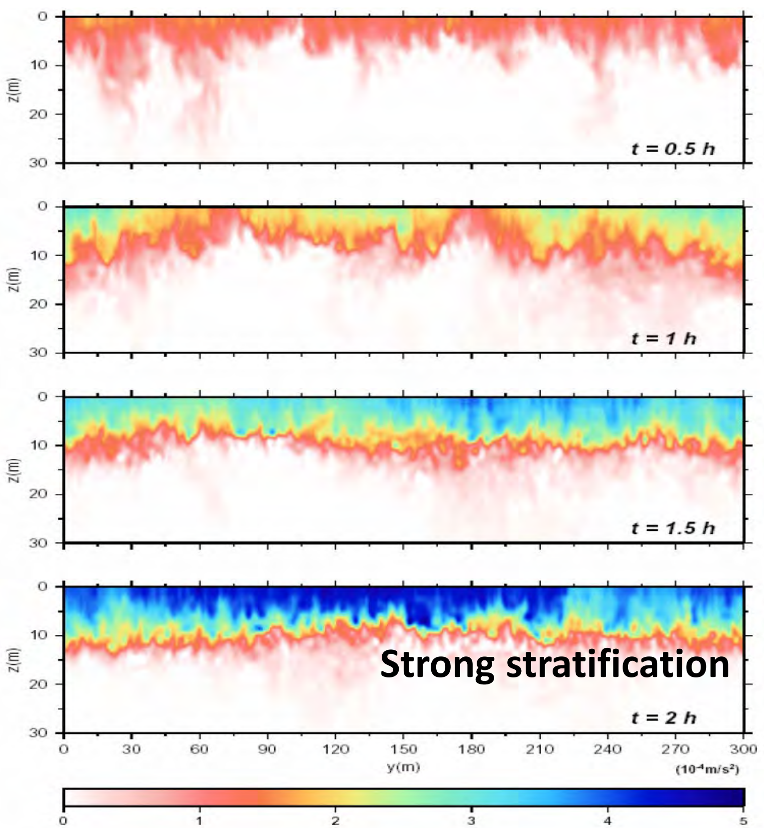
- night - Surface cooling generates turbulence, and deepens the mixed layer depth.
- day - Surface heating suppresses turbulence, and **generates a diurnal thermocline.**

Formation of the Diurnal Thermocline

Observation result (Brainerd and Gregg. 1993)
Evolution of **potential temperature** and **dissipation** during the daytime



LES result (Noh and Goh 2009)
Evolution of **buoyancy** under surface heating



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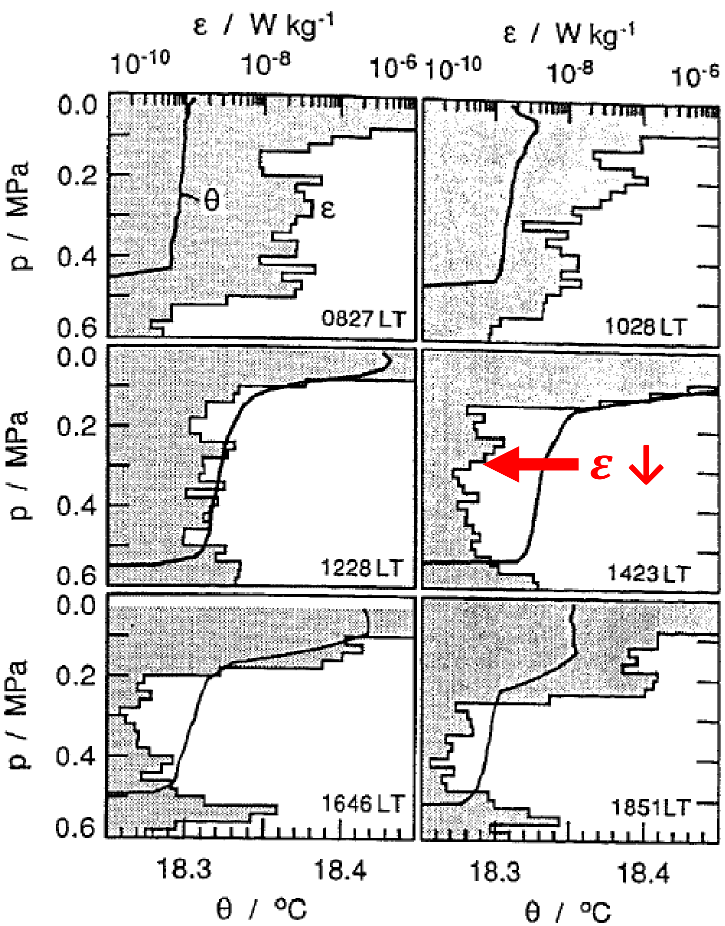

$\frac{d\theta}{dz}$ ↑

ε ↓

LES result (Noh and Goh 2009)
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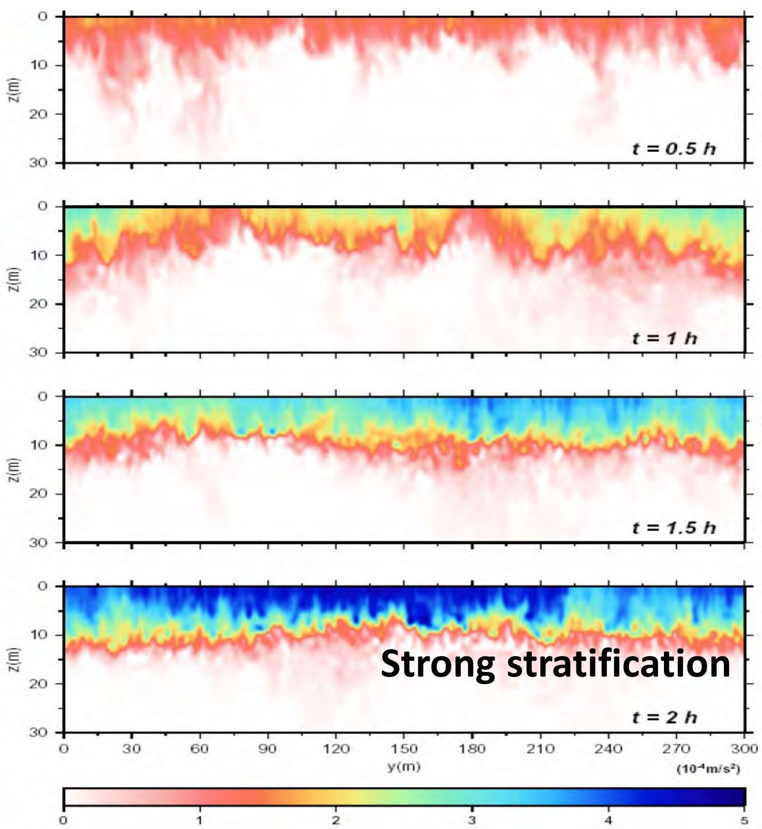
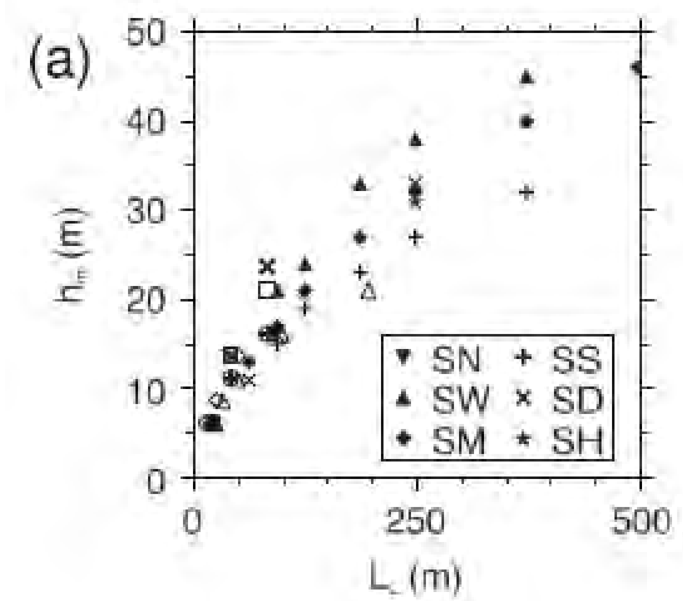
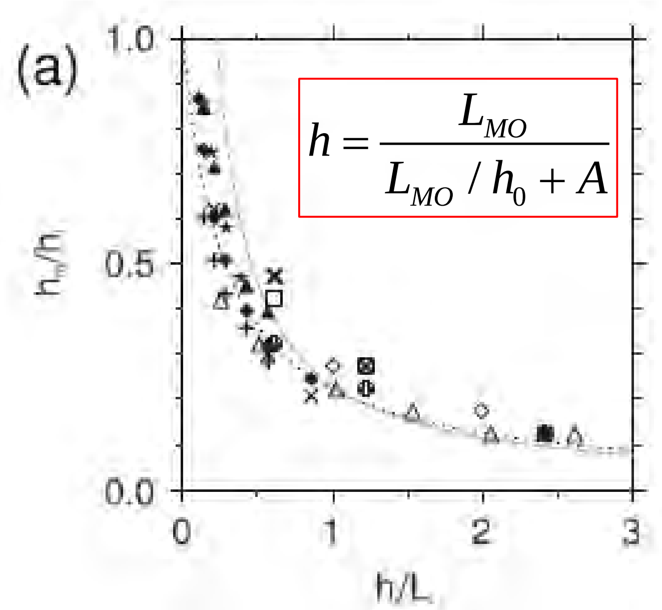


Then, how is the depth of the diurnal thermocline determined?

Langmuir Turbulence and Surface Heating in the Ocean Surface Boundary Layer (Pearson et al., *JPO* 2015)





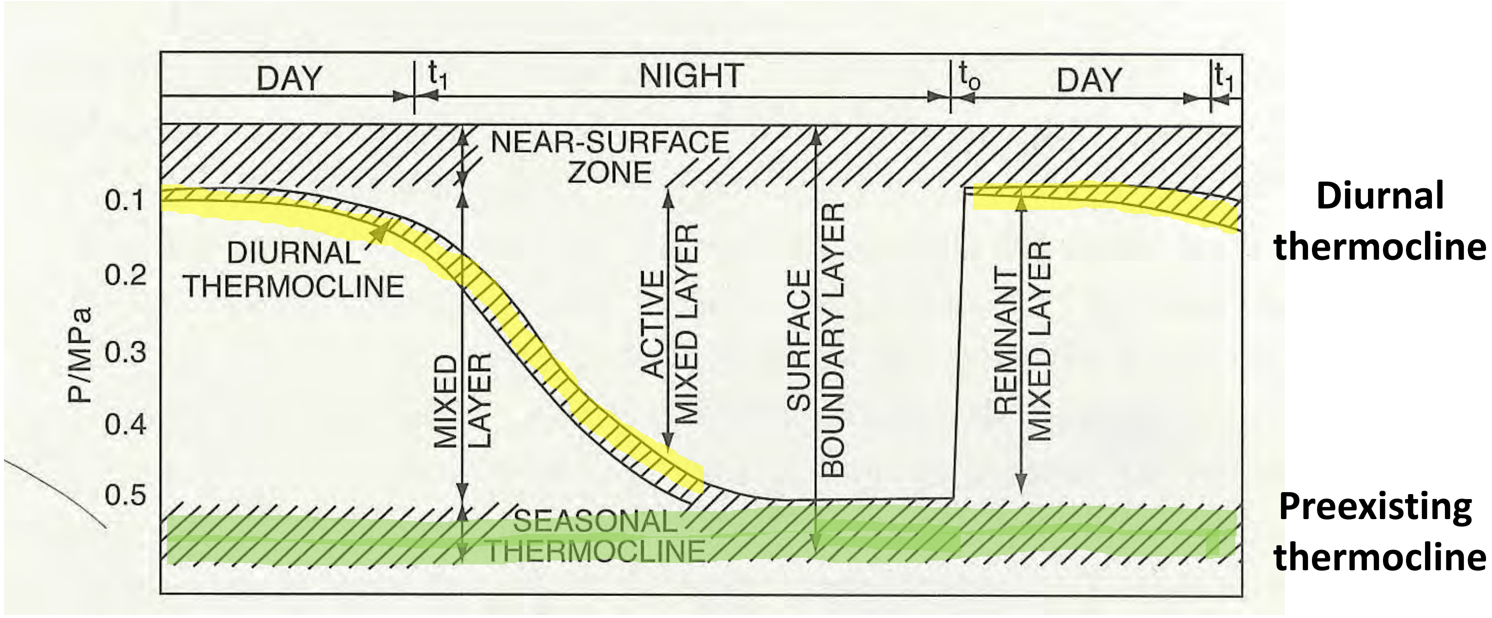
$$h = \frac{L_{MO}}{L_{MO} / h_0 + A}$$

- *h* is scaled by $\boldsymbol{L_{MO}}$.

- The slower increase of *h* than L_{MO} is explained by the effect of the **preexisting thermocline** (h_0).

How is the depth of a diurnal thermocline scaled ?

- Is it scaled by the Monin-Obukhov scale or by the Zilitinkevich scale?
- How is it affected by the preexisting thermocline?



Investigation of the Formation of a Diurnal Thermocline Using LES

- LES model - PALM
- model domain & grid : Lx = Ly = 300 m, H = 80m,

  $\Delta x = \Delta y = \Delta z = 1.25$ m

- forcing
  - **Constant heat flux** : $Q_o(=5.0*10^{-7}m^2s^{-3})$, $0.5Q_o$, $0.25Q_o$
  - **wind stress** : u* = 0.01 m/s
  - **rotation effect** : f = 0.25, 0.5, 1, 1.4 x 10^{-4} s^{-1}
  - LC & WB

- Integration

  - 12 hr spin-up with $Q_0 = 0$
  - from the **homogeneous layer** and the **preexisting thermocline(h_0)**.
  - Stratification below h_0 : N^2=1.0, $5.0\times10^{-4}s^{-1}$

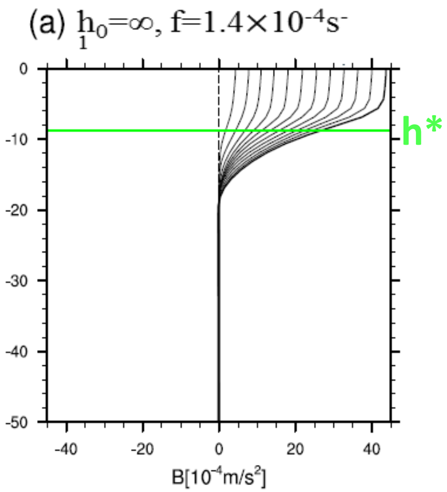
- the definition of h : the depth which has maximum stratification(N^2)

Evolution of buoyancy profile

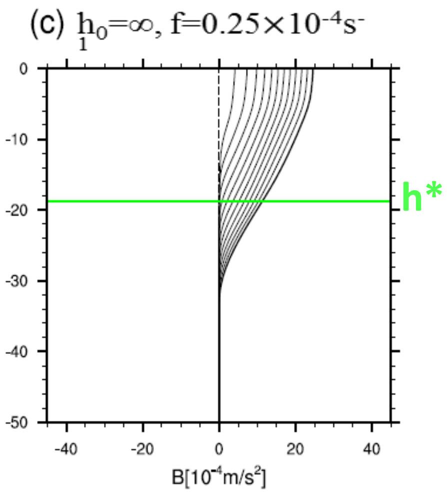
h_0 : depth of the preexisting thermocline
h : MLD with preexisting thermocline
h* : MLD without preexisting thermocline

Preexisting thermocline X

Large f



small f



- The formation of a diurnal thermocline is **strongly affected by f.**

$$\rightarrow \quad h = \frac{L_{MO}}{L_{MO} / h_0 + A}$$ (Pearson et al. 2015)

When $h_0 \rightarrow \infty, \quad h \propto L_{MO} \left(= \frac{u_*^3}{Q_0} \right)$

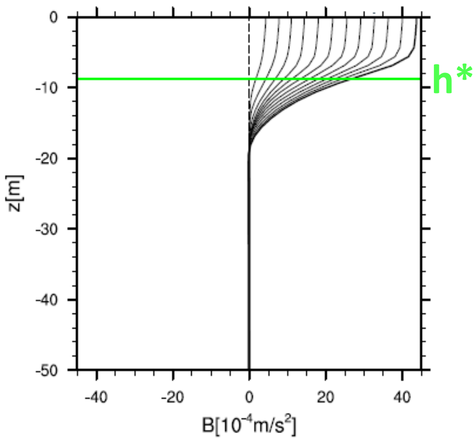
Evolutions of buoyancy profile

Preexisting thermocline X

h_0 : depth of the preexisting thermocline
h : MLD with preexisting thermocline
h* : MLD without preexisting thermocline

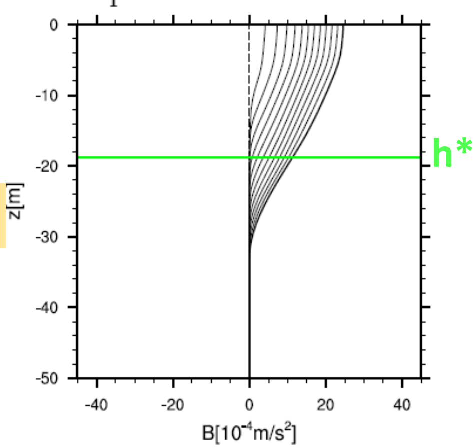
(a) $h_0=\infty$, $f=1.4\times10^{-4}s^{-1}$

Large f



(c) $h_0=\infty$, $f=0.25\times10^{-4}s^{-1}$

small f



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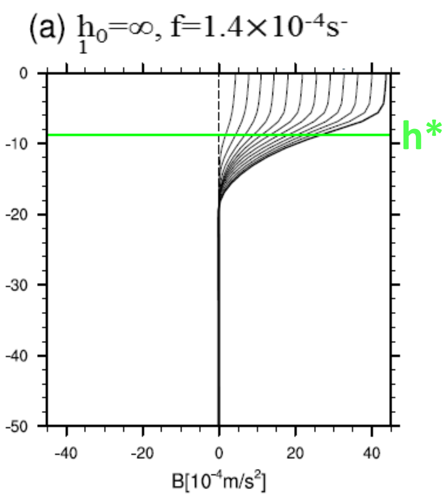
Evolutions of buoyancy profile

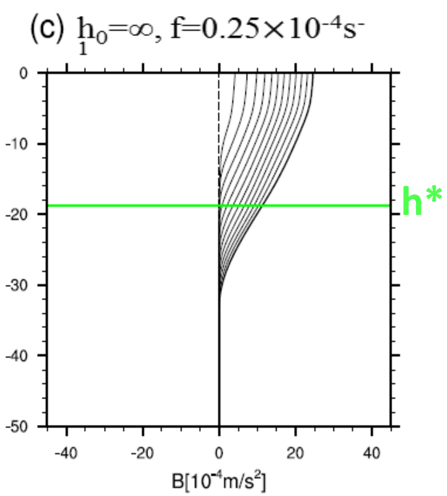
Preexisting thermocline X

h_0 **: depth of the preexisting thermocline**
h : MLD with preexisting thermocline
h* : MLD without preexisting thermocline

Large f



small f

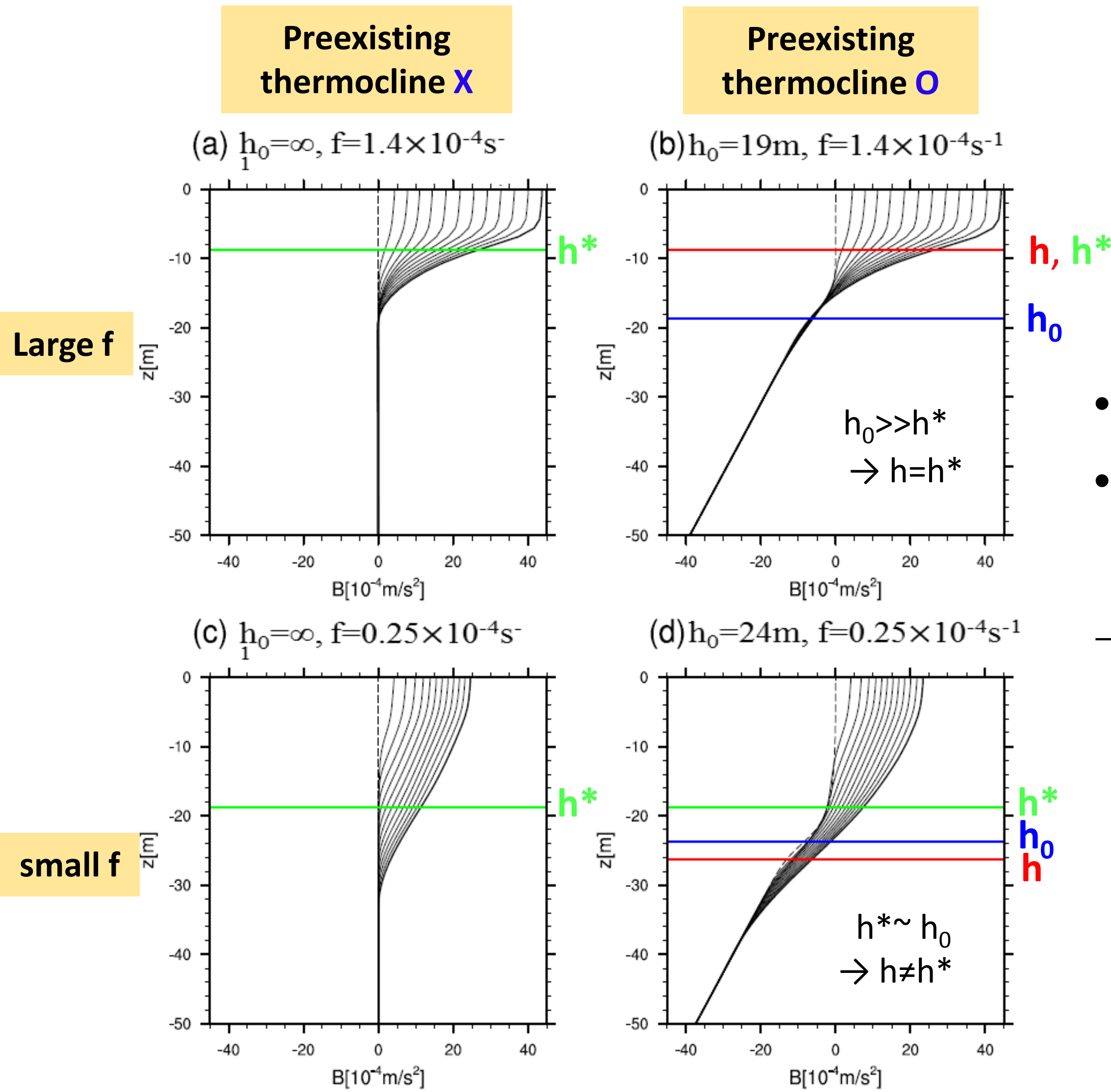


- The formation of a diurnal thermocline is **strongly affected by f.**

$$\rightarrow \quad h = \frac{L_{MO}}{L_{MO}/h_0 + A}$$ is not proper!

When $h_0 \rightarrow \infty, \; h \propto L_{MO} \left(= \frac{u_*^3}{Q_0} \right)$

Evolutions of buoyancy profile



h_0 : depth of the preexisting thermocline

h : MLD with preexisting thermocline

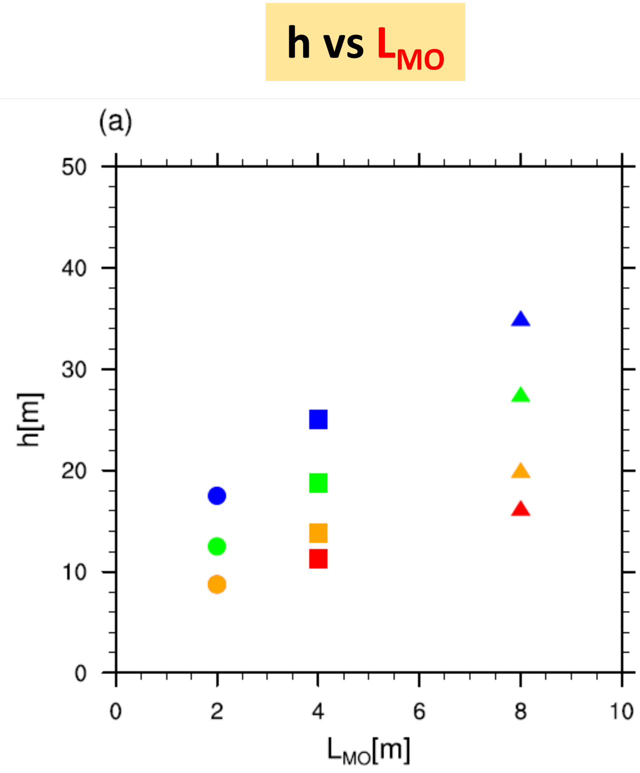
h* : MLD without preexisting thermocline

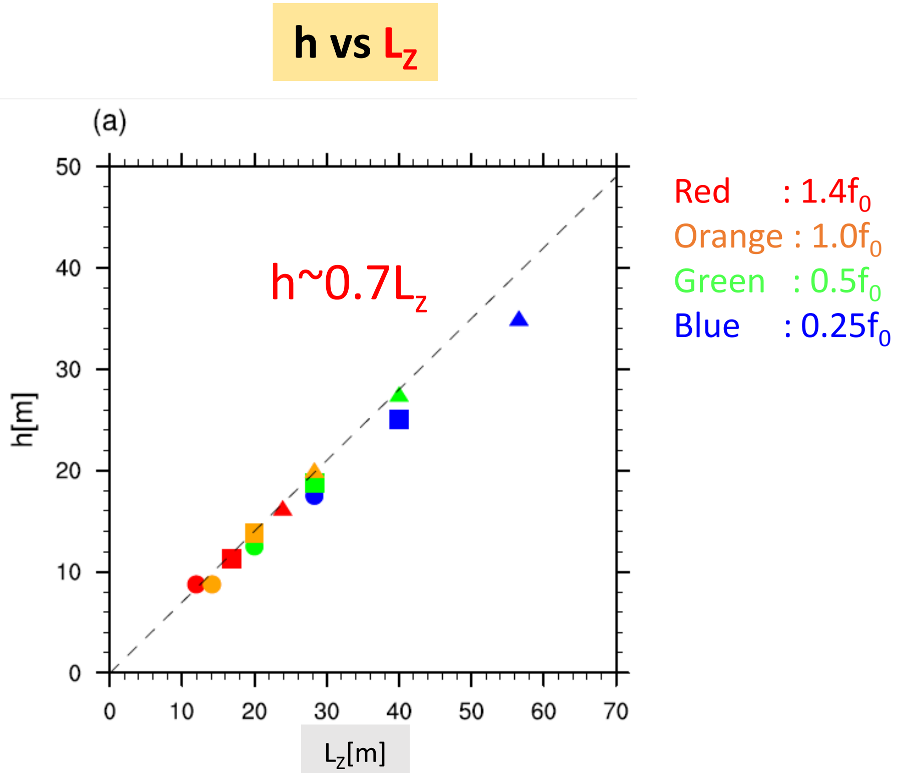
- h is not affected by h_0, if $h_0 \gg h^*$
- h can be larger than h*, when $h^* \sim h_0$.

$$\rightarrow \quad h = \frac{L_{MO}}{L_{MO} / h_0 + A}$$

is not proper!

Scaling of the depth of a diurnal thermocline
(no preexisting thermocline)



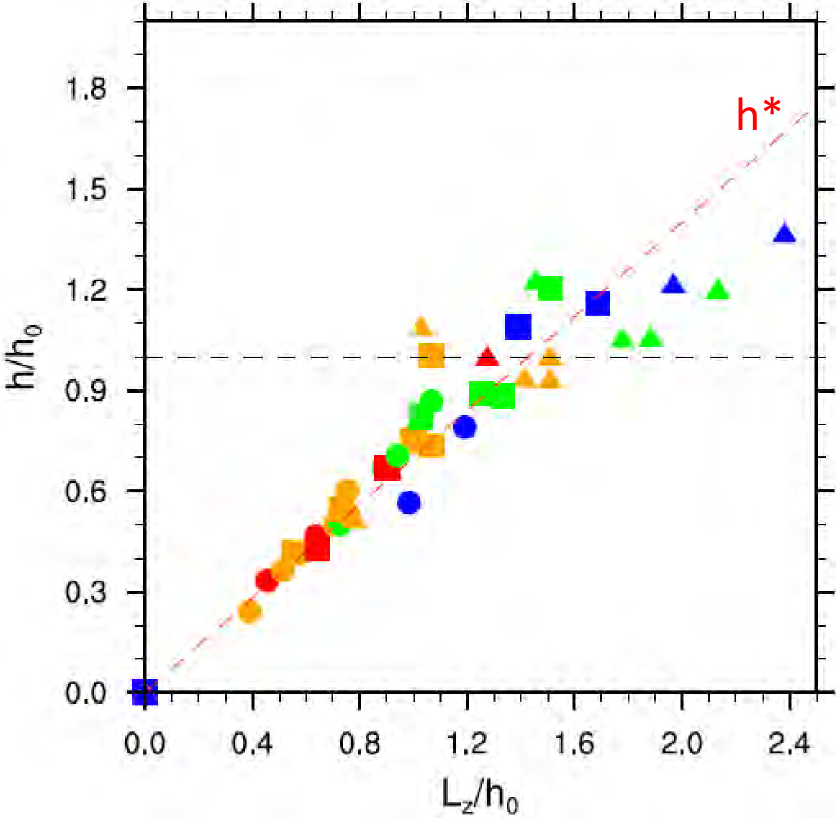


- h should be scaled by L_z, instead of L_{MO} as in the case of a seasonal thermocline.
- h(diurnal thermocline depth) ~ $0.7L_z$

The effect of the preexisting thermocline

h_0 : depth of the preexisting thermocline
h : MLD with preexisting thermocline
h* : MLD without preexisting thermocline

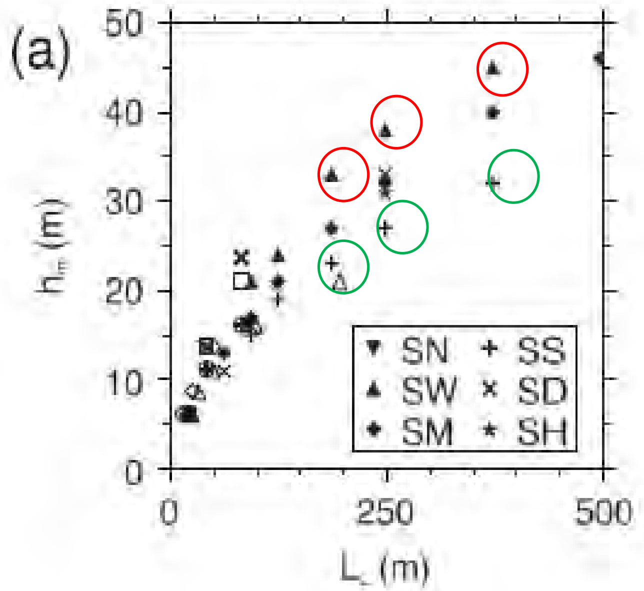
Variation of h/h_0 with L_z/h_0



Red : $1.4f_0$
Orange : $1.0f_0$
Green : $0.5f_0$
Blue : $0.25f_0$

- h is not affected by h_0, when $L_Z/h_0 < 0.9$
- h can be larger than h*, as L_Z/h_0 increases, but ultimately limited by h_0, since stratification suppresses downward heat transport.
- Scatter appears at larger L_Z/h_0

Langmuir Turbulence and Surface Heating in the Ocean Surface Boundary Layer (Pearson et al., *JPO* 2015)



	H_0 (W m^{-2})	L_L (m)	f (s^{-1})	δ (m)	h_i (m)
SS	16–64	248–62	1.4×10^{-4}	4.8	53
SM	8–64	496–62	10^{-4}	4.8	53
SW	16–64	248–62	0.5×10^{-4}	4.8	53
SN	8–64	496–62	0	4.8	53
SD	16, 64	248, 62	10^{-4}	14.4	53
SH	10.6–42.6	372–93	10^{-4}	4.8	36
Neutral	0	∞	10^{-4}	4.8	53

- *When u* and f are constant,* data actually represent the relation $h \propto L_Z \propto Q_0^{-1/2} \leftrightarrow \propto L_{MO}^{1/2}$
- Slower increase of h than L_{MO} is not due to the effect of h_0.

Conclusion

- The depth of a diurnal thermocline(h) should be scaled by L_z, not by L_{MO}.
- *h* is not affected by the preexisting thermocline(h_0), when $L_Z/h_0 < 0.9$.
- *h* can be larger than h_0, when $L_Z/h_0 > 0.9$, but ultimately limited by h_0, since stratification suppresses downward heat transport.