Wave-induced turbulence: theory and practice

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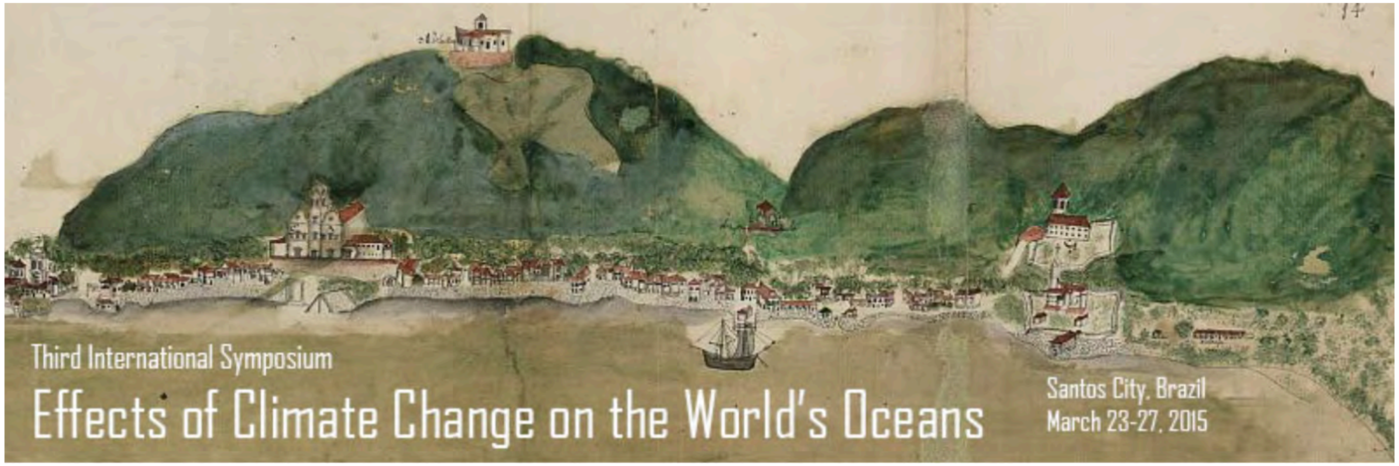
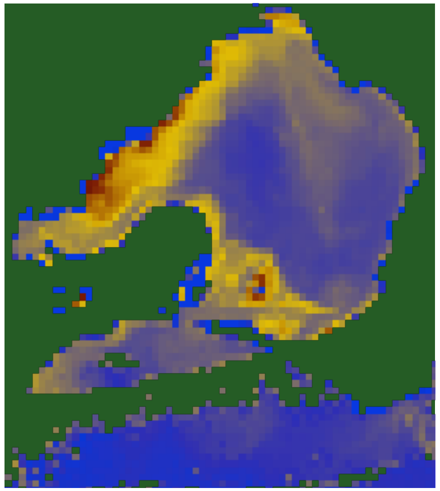

Third International Symposium
Effects of Climate Change on the World's Oceans
Santos City, Brazil
March 23-27, 2015

Motivation

- Marine biogeochemistry and ecosystems are closely connected with the physical processes in the upper ocean
- Changes to the physical environment of the ocean can have impact on ocean biology
- Among such dynamics, ocean mixing is one of the most important
- Until recently, turbulence produced by the orbital motion of surface waves was not accounted for, and this fact limits performance of the models for the upper-ocean mixing and air-sea interactions

Motivation

- Wave-induced mixing
  - dissolved gases
  - nutrients
  - water temperature and stratification
- Direct influence in finite-depths:
  - sediment suspension
  - impact on corals, sea grass



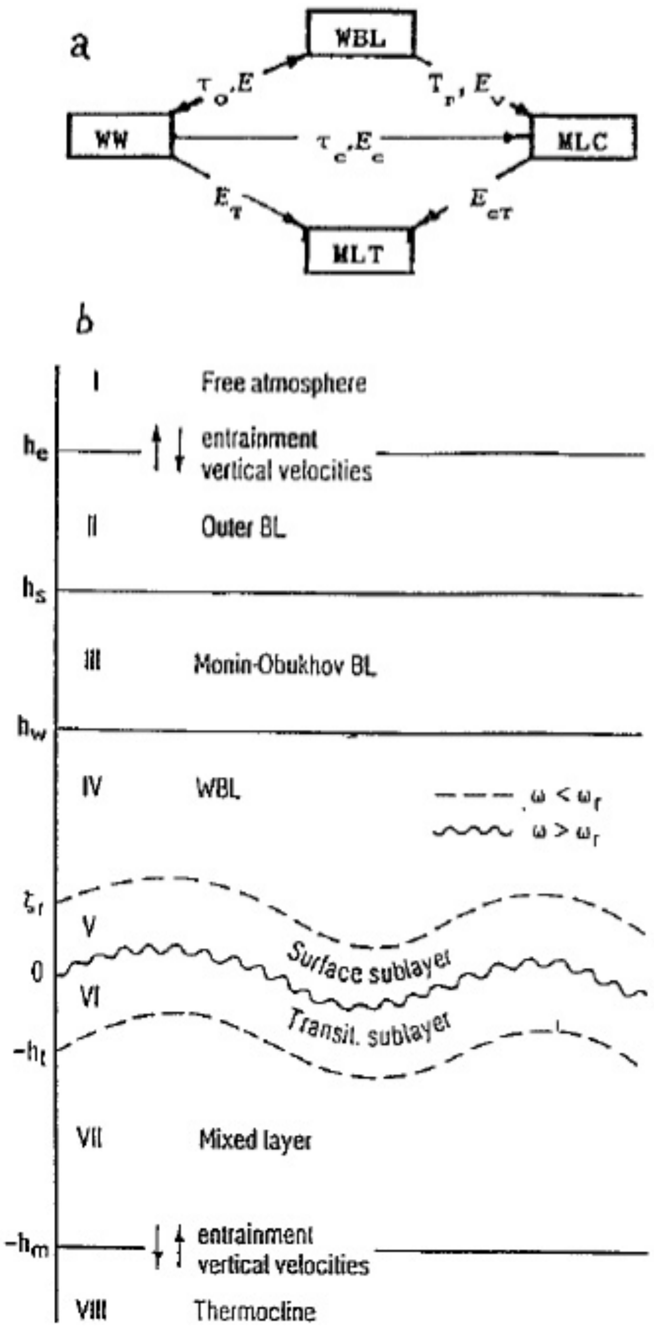
Motivation

**Small- and large-scale air-sea processes are essentially coupled in nature,
but not in the models**

- Atmospheric boundary layer
  - winds generate waves
  - waves provide surface roughness and change the winds
  - waves evolve, fluxes change
- Upper ocean mixed layer
  - waves generate turbulence, moderate and facilitate mixing
  - change the circulation, SST

Tradition and future

- Small scales and large scales are separated. Models reach saturation in their performance
- They need to be coupled, from turbulence to climate. Understanding of geophysics exists, computer capacity exists



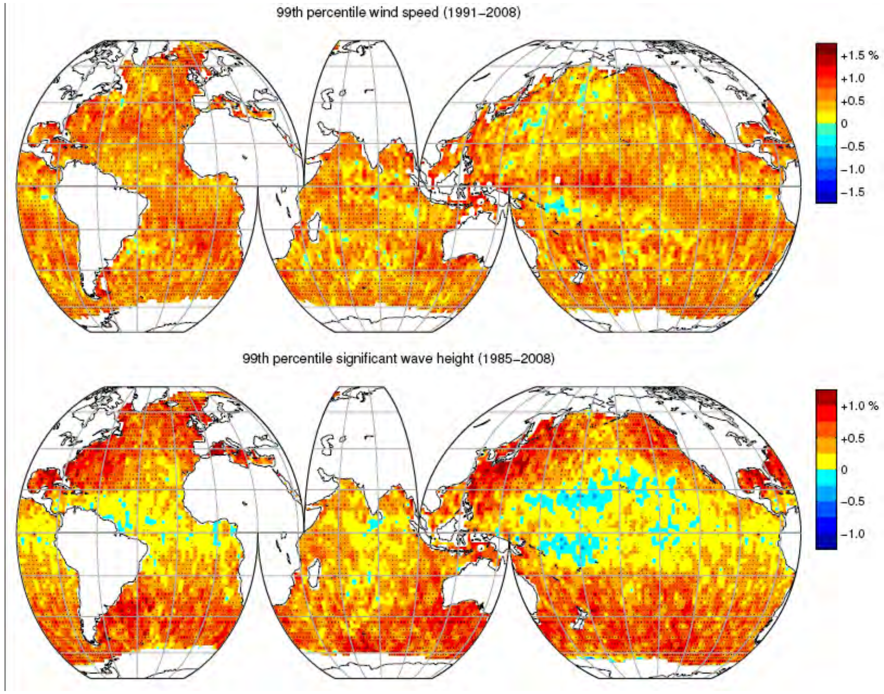
Chalikov and Belevich, BLM, 1993

Motivation

Waves influences the climate, climate affects the waves

Winds and waves change

Observations



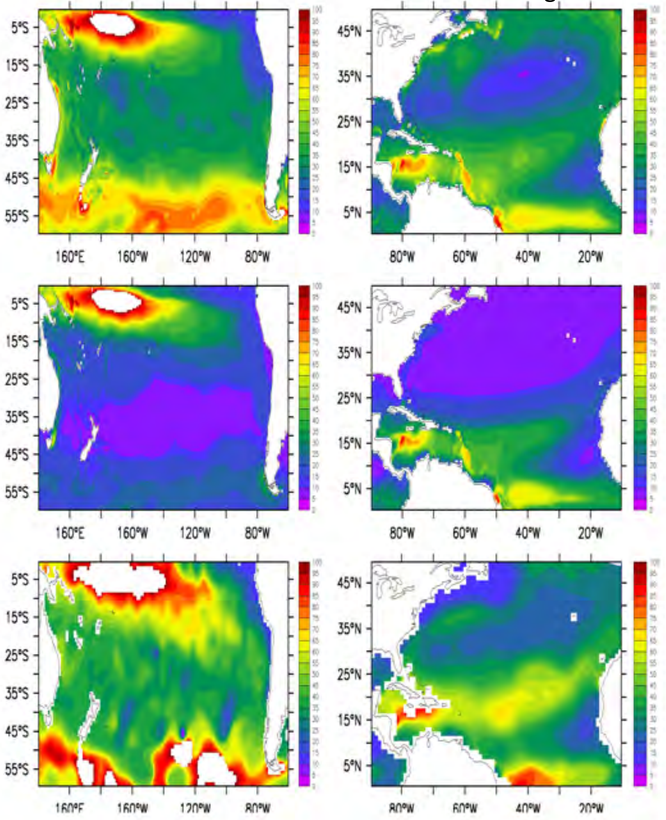
Young et al., Science, 2011

Sea surface temperature changes due to waves

Model

MLD , Southern Pacific in Feb.

MLD, Northern Atlantic in Aug



With waves

Without waves

World Ocean Atlas

Qiao et al., Ocean Dynamics, 2010

Introduction

- in air-sea interaction and ocean-mixing models, the wind stress is usually parameterised to directly drive the dynamics of the upper ocean
- wind provides momentum and energy fluxes to the ocean surface and thus mixes the upper ocean
- dominant part of the wind stress, however, is supported by the flux of momentum from wind to waves
- these waves break, and the breaking is regarded as the main source of the turbulence across the interface
- the turbulence is then diffused down and the mixing is achieved
- if the wave breaking was the only role of the waves in the upper-ocean mixing, such a scheme would perhaps be feasible
- there are, however, two potential problems in such approach

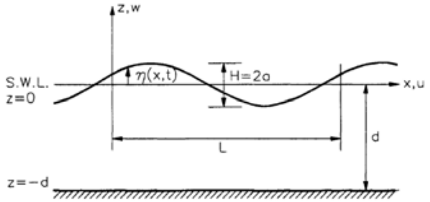
Waves and ocean turbulence

- there are, however, two potential problems in such approach
- first of all, time scales of the turbulence lifetime and turbulence diffusion down to some 100m should agree
- secondly, before the momentum is received by the upper ocean in the form of turbulence and mean currents, it goes through a stage of surface wave motion
- such motion can directly affect or influence the upper-ocean mixing and other processes, and thus ignoring the wave phase of momentum transformation may undermine accuracy and perhaps even validity of such parameterisations
- there are at least two processes in the upper ocean which can deliver turbulence straight to the depth of 100m or so instead of diffusing it from the top
- these are wave-induced turbulence and Langmuir ciruclation
- 2-3m of the ocean water have the same heat capacity as the entire atmosphere

Linear Wave Theory.
Governing equations

$$\varphi(x,z,t) = \frac{ag}{\omega}\frac{\cosh[k(d+z)]}{\cosh[kd]}\cos(kx-\omega t)$$

- Most fluid mechanics problems can be solved by considering the governing Equations of conservation of mass, momentum and energy



Define the velocity potential φ

$$u = -\frac{\partial\varphi}{\partial x}, \quad w = -\frac{\partial\varphi}{\partial z}$$

- Laplace Equation (Continuity Equation) - conservation of mass (two-dimensional case):

$$\frac{\partial^2\varphi}{\partial x^2} + \frac{\partial^2\varphi}{\partial z^2} = 0$$

- Unsteady Bernoulli Equation – conservation of momentum:

$$\frac{p}{\rho} + gz - \frac{\partial\varphi}{\partial t} = 0$$

Kinsman, 1965: Wind Waves

based on Phillips (1961)

Navier-Stokes equation

linearised boundary conditions, with surface tension *T*

$$\frac{\partial u}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\nabla^2 u$$

$$\frac{\partial w}{\partial t} = -\frac{1}{\rho}\frac{\partial p}{\partial z} + \nu\nabla^2 w - g$$

$$\frac{\partial u}{\partial x} + \frac{\partial w}{\partial z} = 0$$

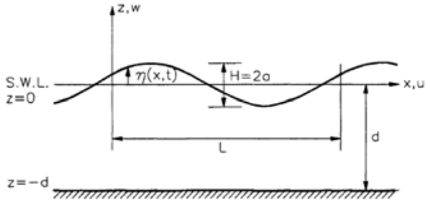
$$\frac{\partial \eta}{\partial t} = w_{z=0}$$

$$p - 2\mu\frac{\partial w}{\partial z} = -\frac{\partial^2 \eta}{\partial x^2}\mathrm{T}_{z=\eta}$$

$$\frac{\partial w}{\partial x} + \frac{\partial u}{\partial z} = 0_{z=\eta}$$

Solutions

vorticity



$$\omega = \frac{\partial w}{\partial x} - \frac{\partial u}{\partial z} = \nabla^2\Psi$$

$$\omega = \beta\frac{i\sigma}{\nu}e^{mz}e^{i(kx+\sigma t)} =$$

$$= -2\gamma k\sigma \exp\left(\sqrt{\frac{\sigma_{real}}{2\nu}}z - \frac{2\sigma_{real}}{\mathrm{Re}_w}\right)\exp\left\{i\left(kx + \sqrt{\frac{\sigma_{real}}{2\nu}}z + \sigma_{real}t\right)\right\}$$

$$\frac{\delta_z}{\lambda} = \frac{1}{\lambda}\sqrt{\frac{2\nu}{\sigma_{real}}} = \frac{1}{2\pi}\sqrt{\frac{2\nu k^2}{\sigma_{real}}} = \frac{\sqrt{2}}{2\pi}\frac{1}{\sqrt{\mathrm{Re}_w}}$$

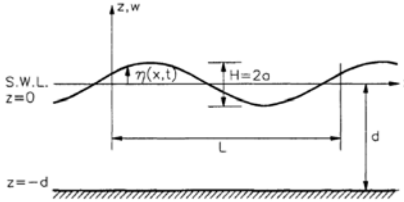
- exponential decay in *z* and *t*
- oscillations in *x*, *z* and *t*
- ‘length’ of vertical vorticity oscillation is much smaller than *λ*

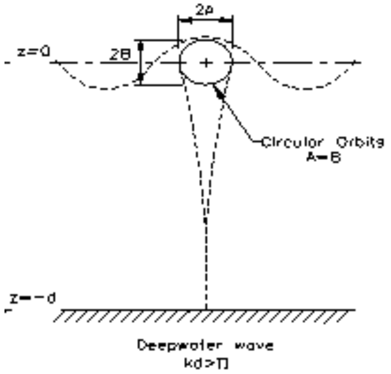
Hypothesis of the Wave Reynolds Number

Babanin, GRL, 2006

$\eta(x,t) = a_0 \cos(\omega t + kx)$

$a(z) = a_0 \exp(-kz)$





It is the hypothesis that the a-based Reynolds number

$$\mathrm{Re} = \frac{aV}{\nu} = \frac{a^2\omega}{\nu}$$

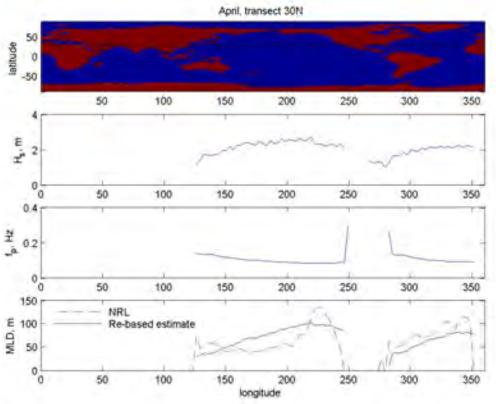
where V=ωa is orbital velocity, and ν is kinematic viscosity of the ocean water, indicates transition from laminar orbital motion to turbulent

Critical Reynolds Number for the Wave-Induced Motion, and Depth of the Mixed Layer

$$\mathrm{Re}(z) = \frac{\omega}{\nu} a_0^2 \exp(-2kz) = \frac{\omega}{\nu} a_0^2 \exp\left(-2\frac{\omega^2}{g} z\right)$$

$$z_{cr} = -\frac{1}{2k} \ln\left(\frac{\mathrm{Re}_{cr}\,\nu}{a_0^2\omega}\right) = \frac{g}{2\omega^2} \ln\left(\frac{a_0^2\omega}{\mathrm{Re}_{cr}\,\nu}\right)$$

Re_{cr}=3000



Dai et al., JPO, 2010

Laboratory Experiment, First Inst. of Oceanography, China

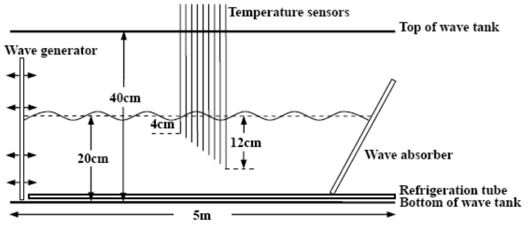


Figure 1. Sketch of the laboratory setup.

Mixing the stratified fluid

experiment (left), model (right)

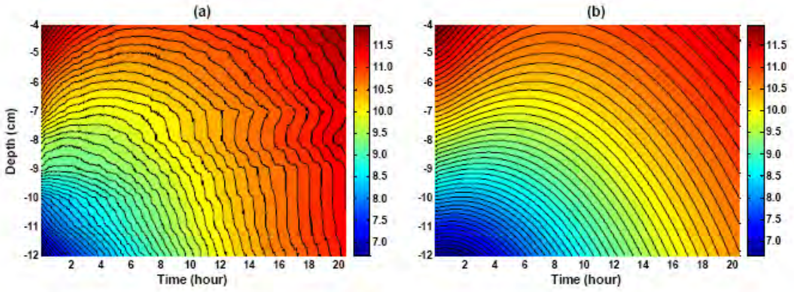
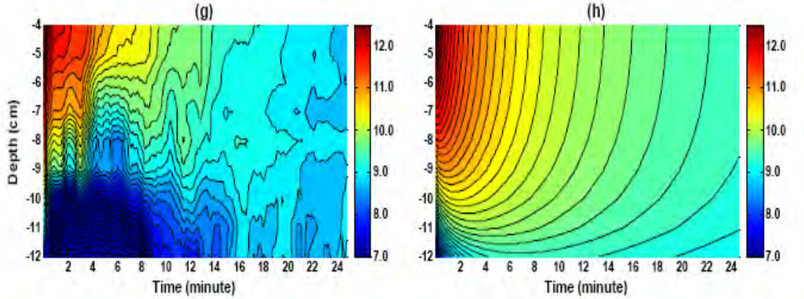


Figure 2. Evolution of the water-temperature profile without waves. (a) observations; (b) numerical simulation with the one-dimensional model. The time is in hours.

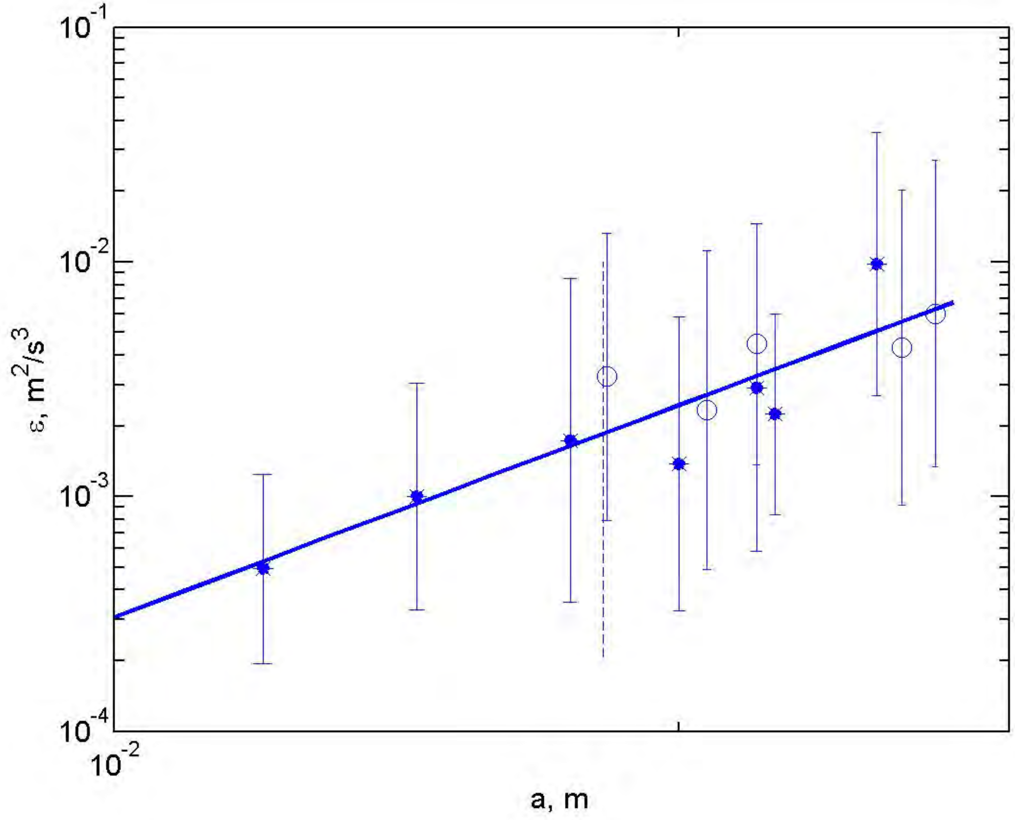


no waves
time scale: hours

non-breaking waves
time scale: minutes

Babanin & Haus, JPO, 2009

Laboratory Experiment, ASIST, RSMAS, University of Miami



$$\varepsilon = 300 \cdot a^{3.0 \pm 1.0}$$

This is close to the expectation: since the force due to the turbulent stresses is proportional to a^2, the energy dissipation rate should be ~a^3.

Model of generation of turbulence in potential waves

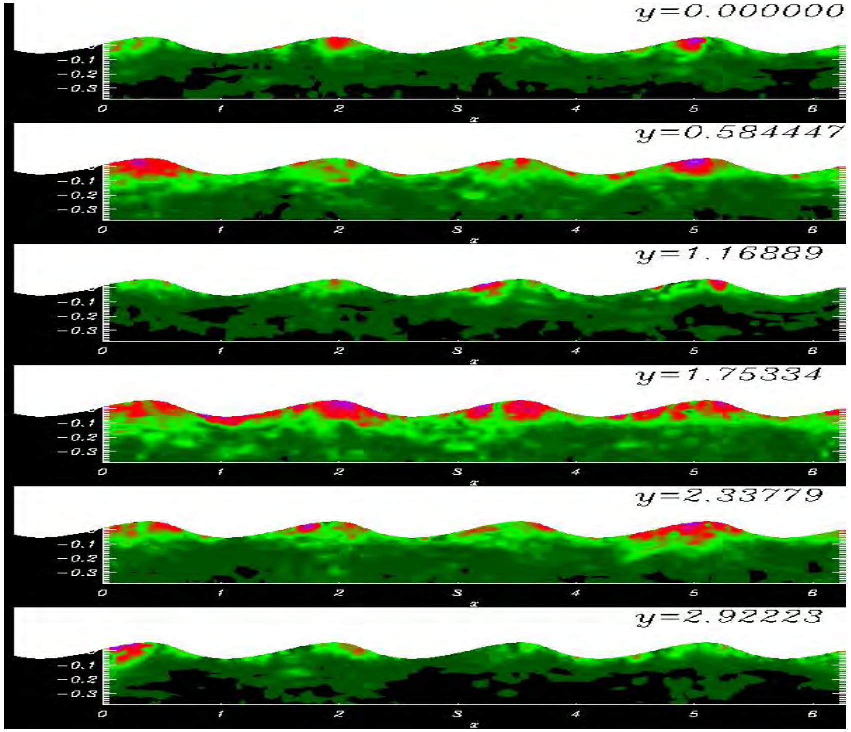
- Regardless of the turbulence source, 3D turbulence is unstable to 2D wave orbital motion *(Benilov, JGR, 2012)*
- Model is based on exact 2-D (x-z) model of surface waves coupled with 3-D LES (x-y-z) model of vortical motion based on Reynolds equation with parameterised subgrid turbulence
- Both systems of equations are written in conformal cylindrical surface-following coordinates
- The one-way coupling of models occurs through components of potential orbital velocity and vorticity components

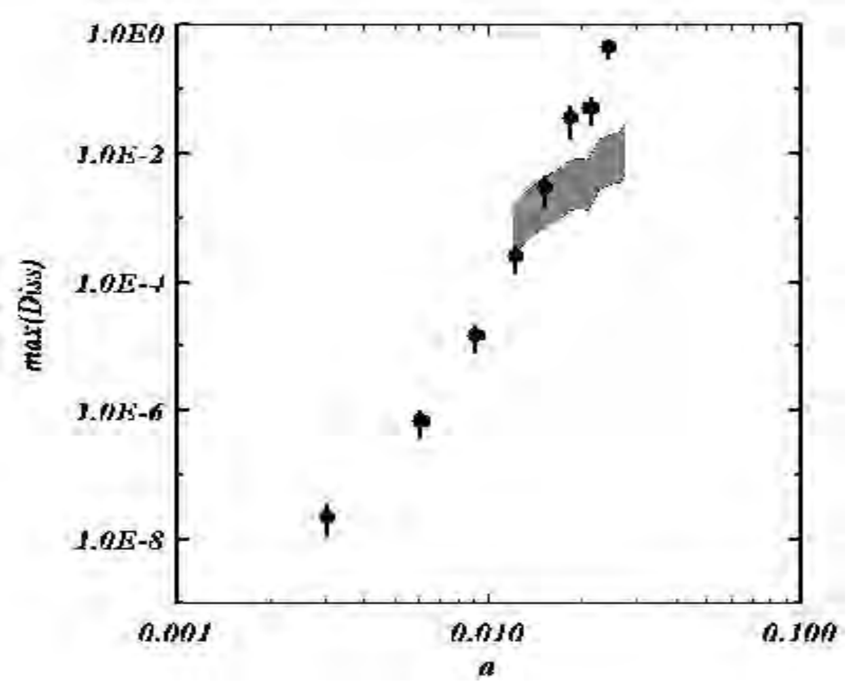
Babanin & Chalikov, JGR, 2012

Babanin & Chalikov, JGR, 2012

Model of generation of turbulence by nonlinear waves

Model is based on exact 2-D (x-z) model of surface waves coupled with 3-D LES (x-y-z) model of vortical motion based on Reynolds equation with parameterised subgrid turbulence





Babanin, 2011, CUP

Swell attenuation

$$\varepsilon = 300 \cdot a^{3.0 \pm 1.0} \quad b = b_1 k \omega^3 = 30. \quad b_1 = 0.004$$

Dissipation

$$\epsilon_{dis} = b_1 k \omega^3 a_0^3 = 0.004 k u_{orb}^3.$$

- volumetric

$$D_a = b_1 k \int_0^\infty u(z)^3 dz = b_1 k u_0 \int_0^\infty \exp(-3kz) dz = \frac{b_1}{3} u_0^3.$$

- per unit of surface

$$D_x = \frac{1}{c_g} D_a = \frac{b_1}{3} 2 \frac{k}{\omega} u_0^3 = \frac{2}{3} b_1 k \omega^2 a_0^3 = \frac{2}{3} b_1 g k^2 a_0^3.$$

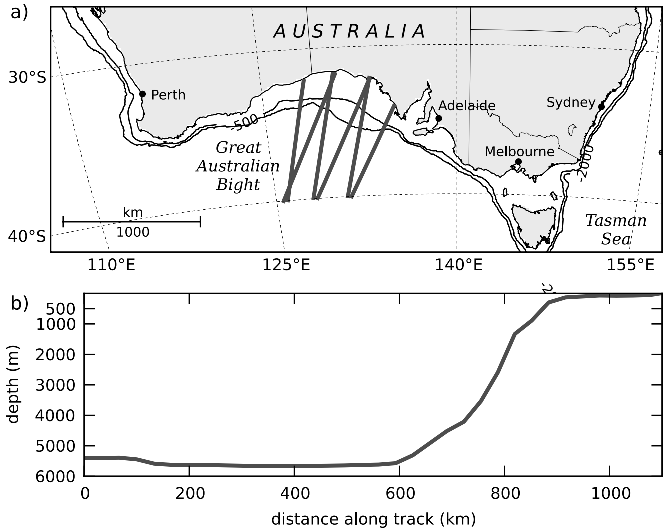
- per unit of propagation distance

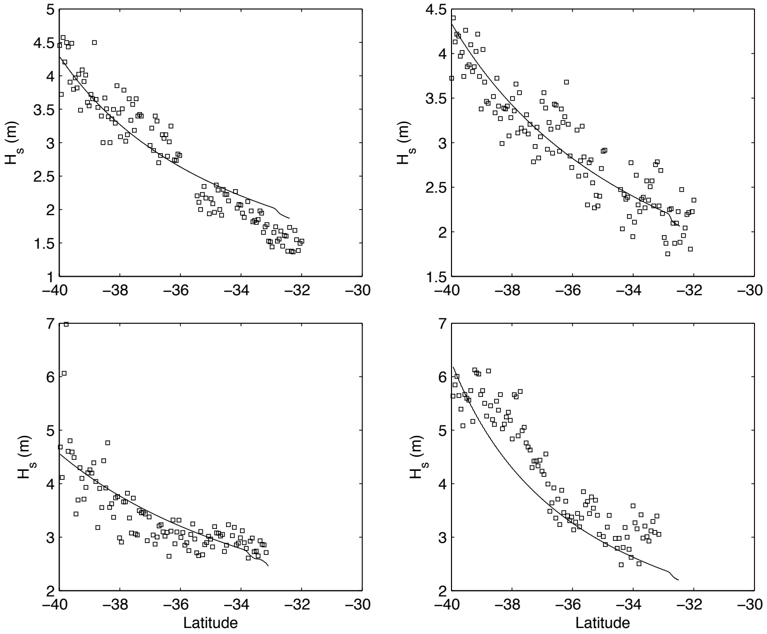
$$\frac{g}{2} \frac{\partial (a_0(x)^2)}{\partial x} = \frac{2}{3} b_1 g k^2 a_0(x)^3,$$

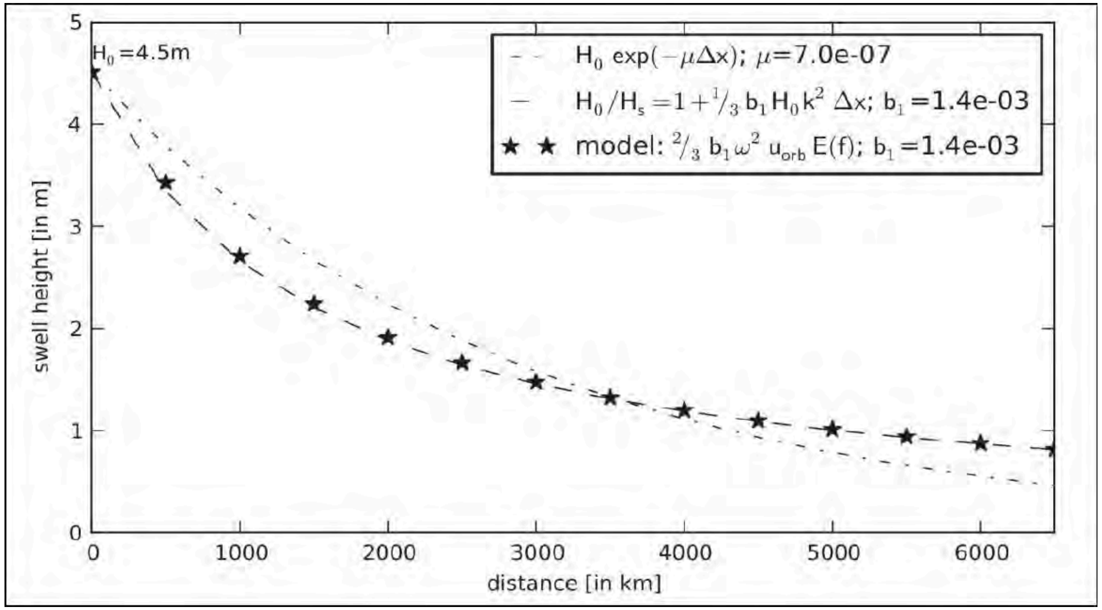
$$a_0(x)^2 = \frac{4}{B^2} x^{-2} = \frac{9}{4 \cdot b_1^2 k^4} x^{-2} = \frac{9}{64} 10^6 k^{-4} x^{-2}.$$

Young, Babanin, Zieger, JPO, 2013

Swell attenuation

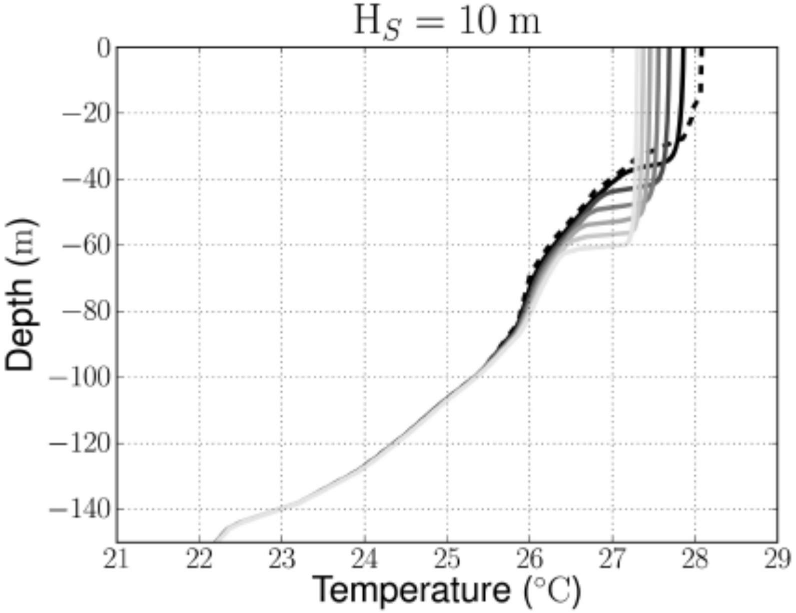


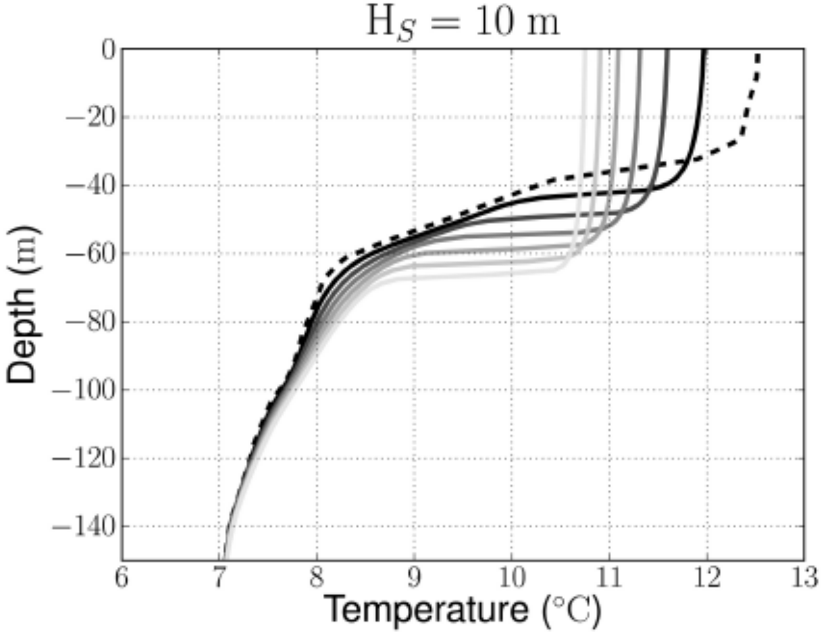




Ghantous and Babanin, Nonlin. Proc. in Geophysics, 2014

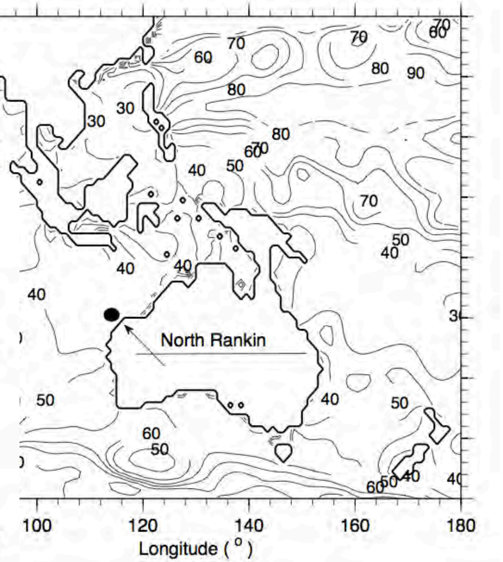
Modelling SST and MLD at the scale of tropical cyclone



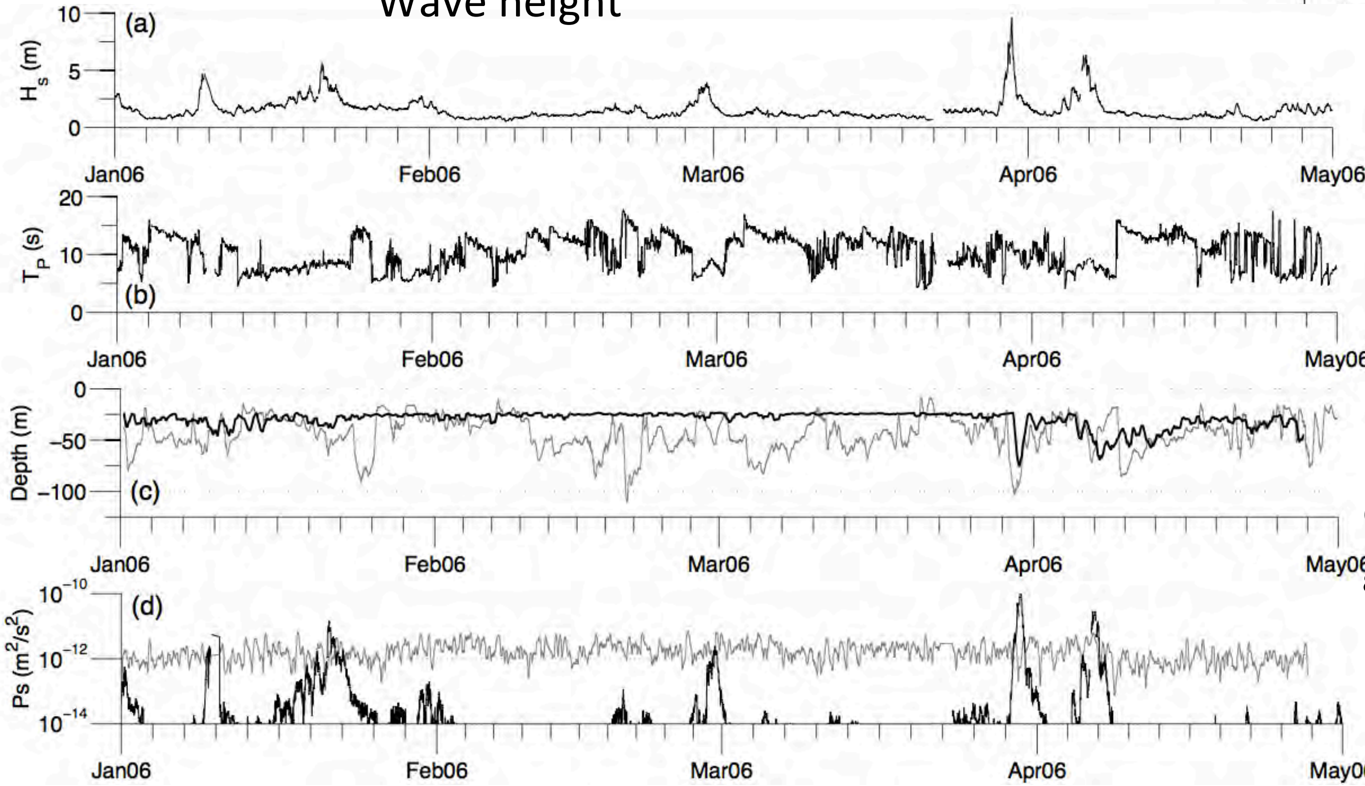


Toffoli et al., JGR, 2012

Field observations, North Rankin mixed layer deepening



Wave height



MLD:
dark – wave induced
grey - measured

Pleskachevski et al., JPO, 2011

Field observations, North Sea, sediment suspension

$$\frac{\partial K}{\partial T_{mean}} + U_i \frac{\partial K}{\partial X_i} = D_K + P_S + G - E_K$$

TKE evolution equation

$$P_s = P_{CURR} = \upsilon_t M_{CURR}^2$$

TKE production

$$M = \partial \bar{u}_i / \partial x_j$$

shear frequency

$$P_s = (\upsilon_{CURR} + \upsilon_{wave})(M_{CURR}^2 + M_{wave}^{AM\ 2})$$

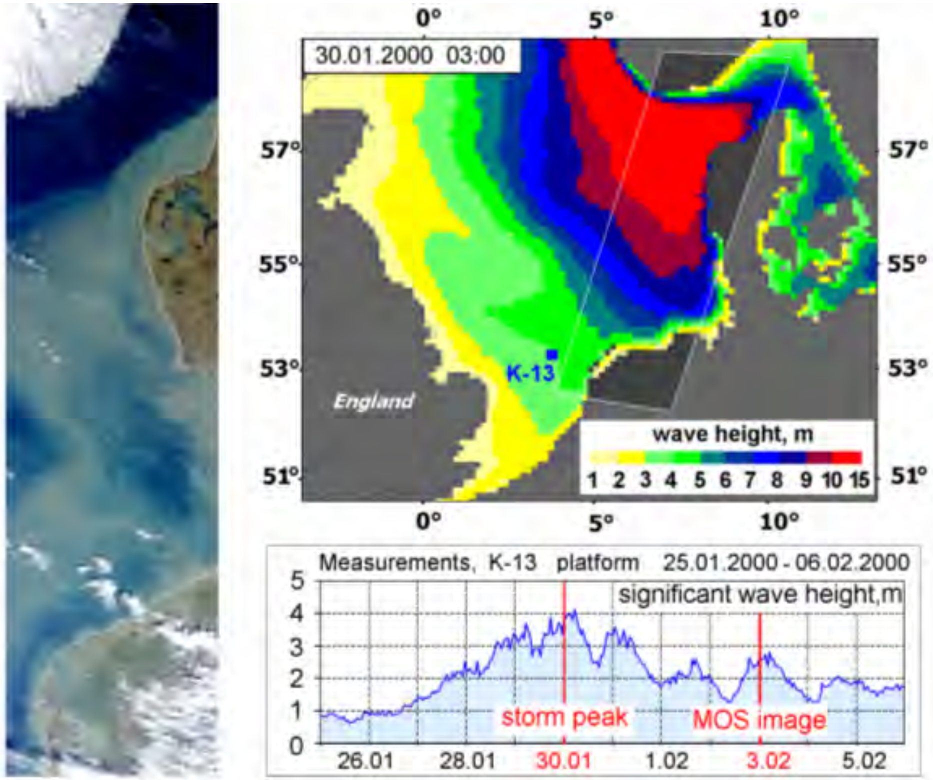
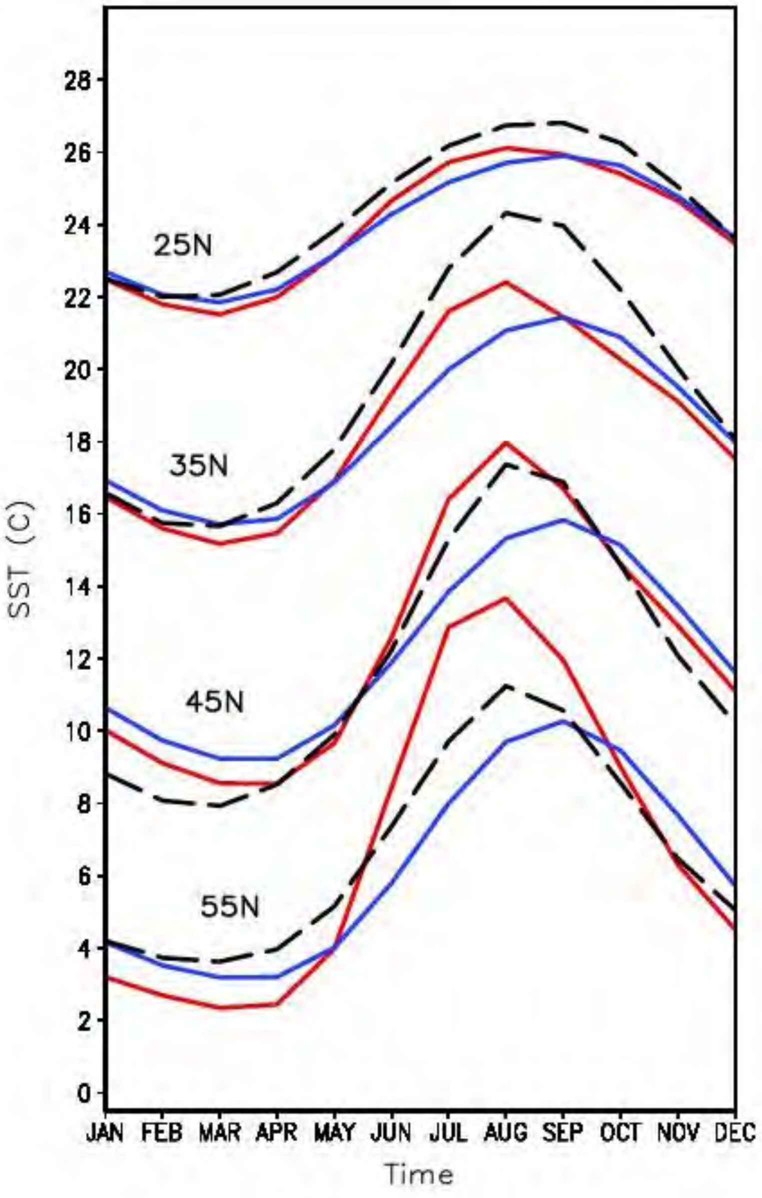


FIG.1. Storm events in the North Sea at 29.01-04.02.2000 (the storm peak on 30.01.2000, at about 03:00 UTC). Optical MOS image of German Bight on 03.02.2000 (left) and significant wave height in the North Sea at the storm peak (right).

Implementing wave-induced mixing in CLIMBER



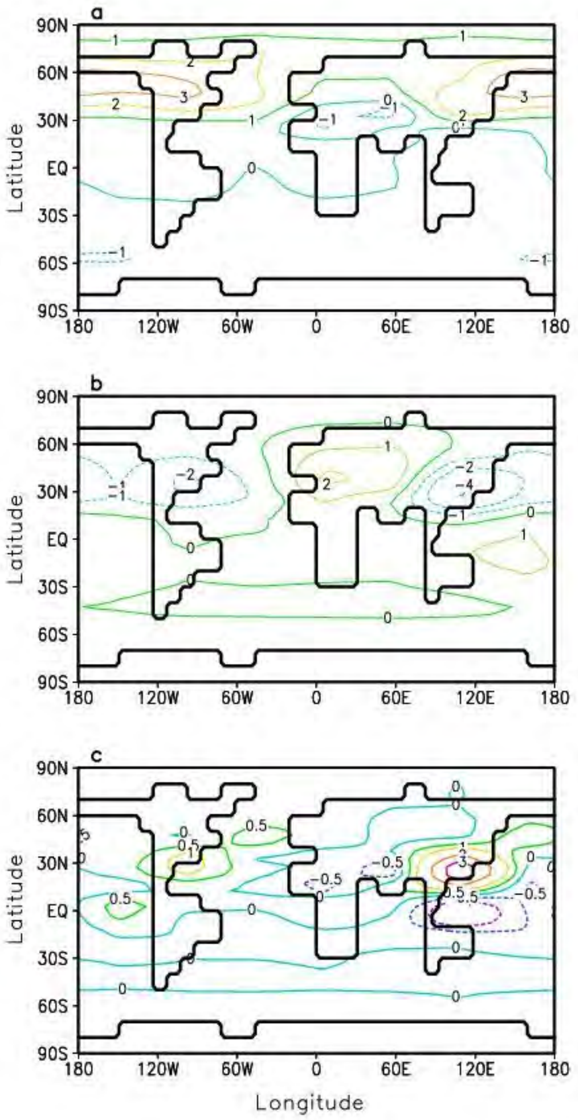
- Seasonal trend of the global zonally averaged SST. Panels shown: 25, 35, 45 and 55 degrees North (from top to bottom). Lines shown: default version of CLIMBER (blue), variable MLD (red) and observations based on Levitus data (black).

- effect is essential outside the tropical areas

- both magnitudes and phases of SST are imporved

Babanin, Ganopolski & Phillips, 2009, Ocean Modelling

Implementing wave-induced mixing in CLIMBER



Global distribution (Northern summer)

- temperature (*degrees*)
- pressure (*mbar*)
- precipitation (*mm* per day)

Conclusions

- marine biogeochemistry and ecosystems are connected with the physics and dynamics of the ocean
- coupling of small-scale models (waves, turbulence) with large-scale models (weather, climate) is necessary
  - physics is continuous
  - computing capabilities allow the coupling
- waves provide feedback
  - to the atmospheric boundary layer
  - to the upper ocean (usually overlooked)
  - to the large-scale air-sea interactions
- wave climate also changes