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Extreme sea level statistics along Estonian coast

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Outline

- Data and background information
- Extreme sea level statistics
- Statistically independent maximum sea levels
- Sea level maxima distribution
- Conclusions

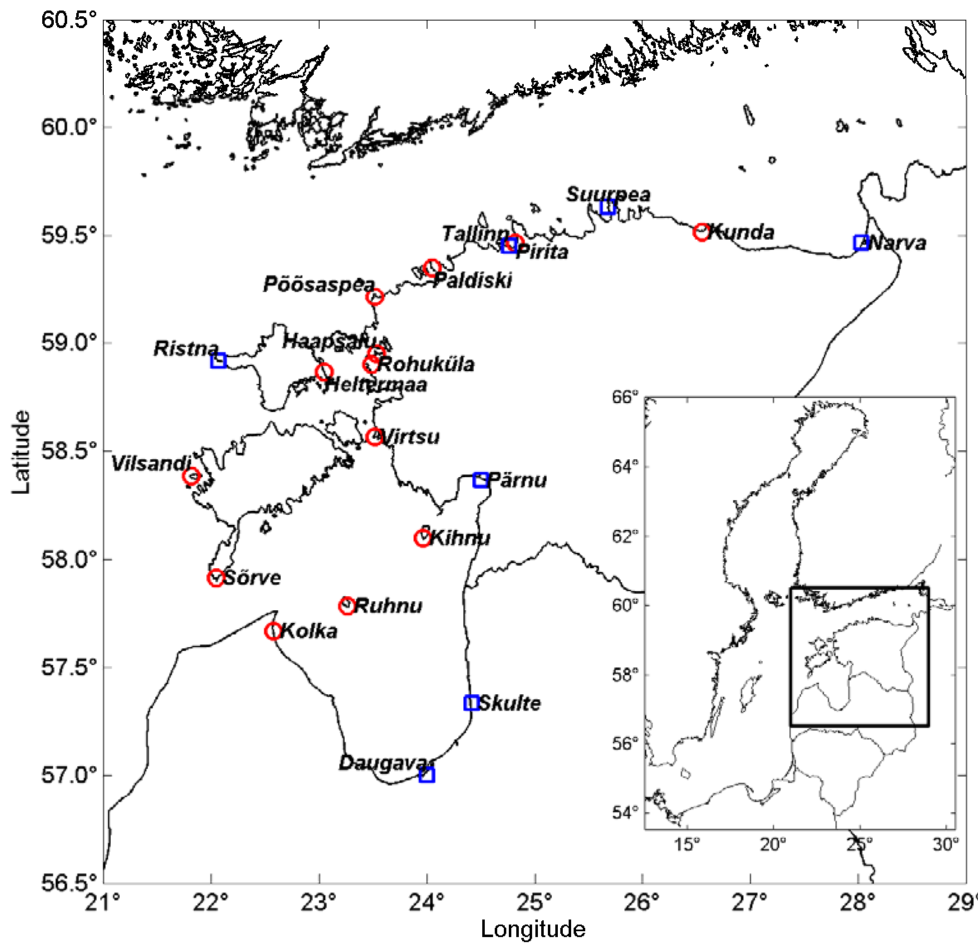
Data and background information

Description:

a) 20 stations including
   7 in Gulf of Finland
   8 in Gulf of Riga
   5 in Väinameri

b) 5 year period

c) Mareograph (blue boxes) or bench stick (red circles)



Data and background information

Measurements 1978-1982 informative table:

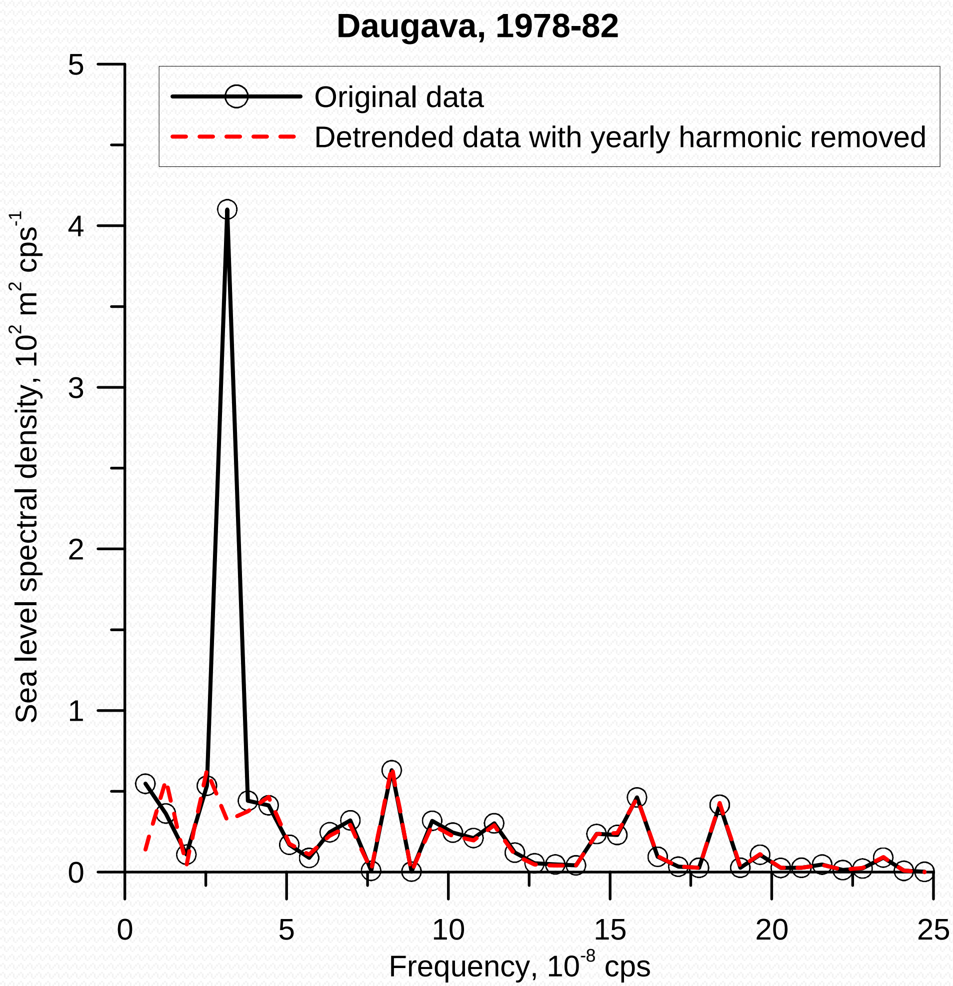
Site		Lat.	Lon.	Data number, *N*	Recording lag, Δt (h)	Missing data number	Mean (cm)	Rise rate (cm/yr)
Gulf of Finland	Narva	59°28'N	28°02'E	43785	1	39	4.3	2.8
	Kunda	59°31'N	26°33'E	7271	6	33	-2.4	2.5
	Pirita	59°28'N	24°49'E	2946	12	706	-	-
	Tallinn	59°27'N	24°46'E	43807	1	17	-2.1	2.7
	Paldiski	59°21'N	24°03'E	6526	6	778	-2.1	2.3
	Põõsaspea	59°13'N	23°31'E	3636	12	16	-2.2	2.4
	Suurpea	59°38'N	25°41'E	40999	1	2825	-2.6	2.1
Väinameri	Haapsalu	58°57'N	23°32'E	3581	12	71	-1.2	3.1
	Rohuküla	58°54'N	23°35'E	3650	12	2	-0.5	3.1
	Heltermaa	58°52'N	23°03'E	7303	6	1	-2.7	2.1
	Ristna	58°55'N	22°04'E	43809	1	15	-5	2.3
	Virtsu	58°34'N	23°31'E	7304	6	0	-2.5	2.3
Gulf of Riga	Pärnu	58°22'N	24°30'E	43682	1	142	6	3.2
	Kihnu	58°06'N	23°58'E	7304	6	0	2.6	1.7
	Skulte	57°20'N	24°30'E	43598	1	226	3.8	2.8
	Daugavgriva	57°00'N	24°00'E	43824	1	0	10.9	2.3
	Kolka	57°40'N	22°40'E	6723	6	581	1.5	1.3
	Sõrve	57°55'N	22°03'E	7252	6	52	-0.8	2
	Ruhnu	57°47'N	23°16'E	7225	6	79	-1.8	1.8
	Vilsandi	58°23'N	21°49'E	4378	6	6	-	-

Data and background information

- Spectra at Daugavgriva station:

The linear trend and annual harmonic, expressed by the peak in the figure, are removed from the original sea level data series.

The sea level residuals are further considered as stationary (homogenous in time) series.

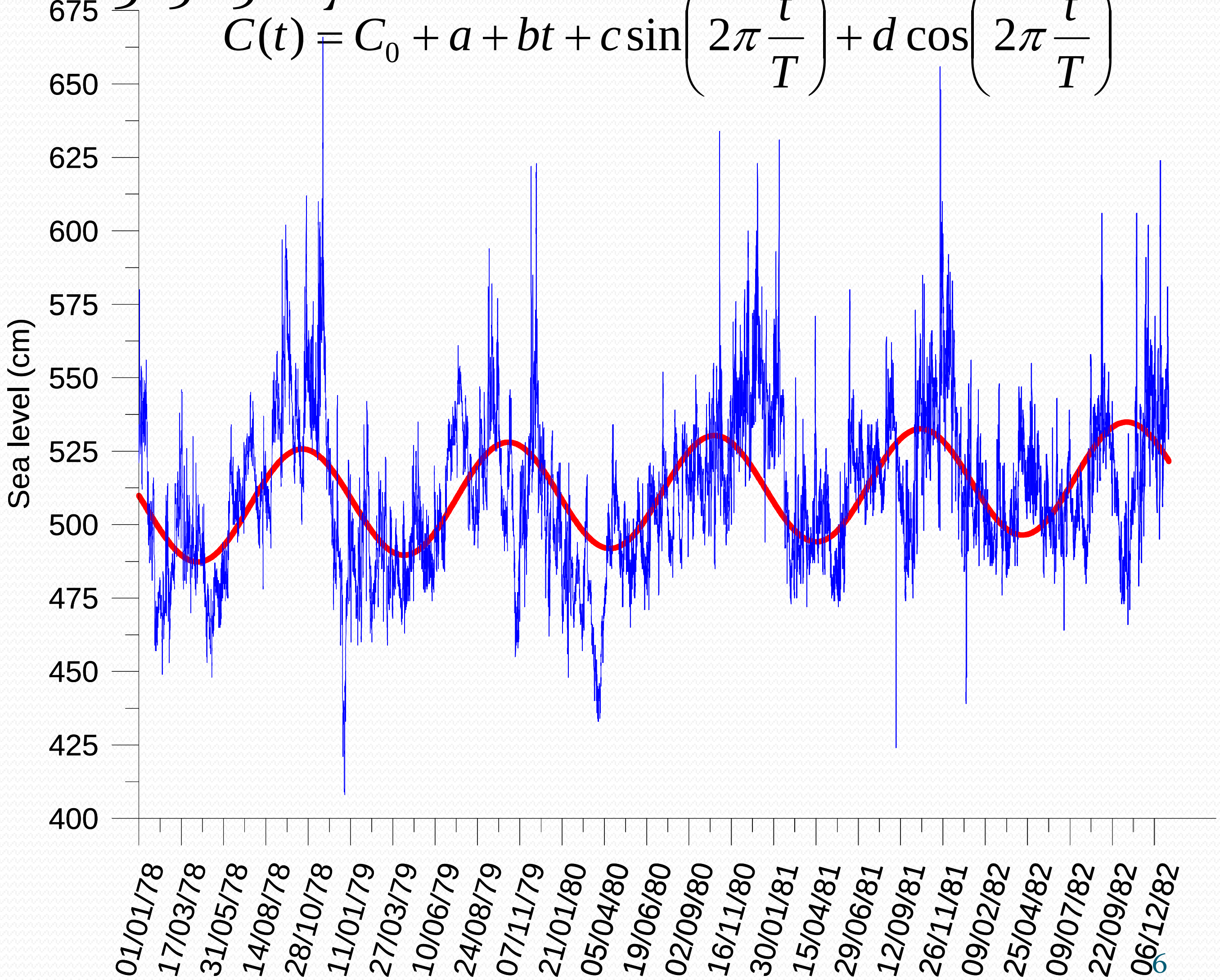


Data and background information

- Water level at Daugavgriva station: harmonic with T = 365.25 days

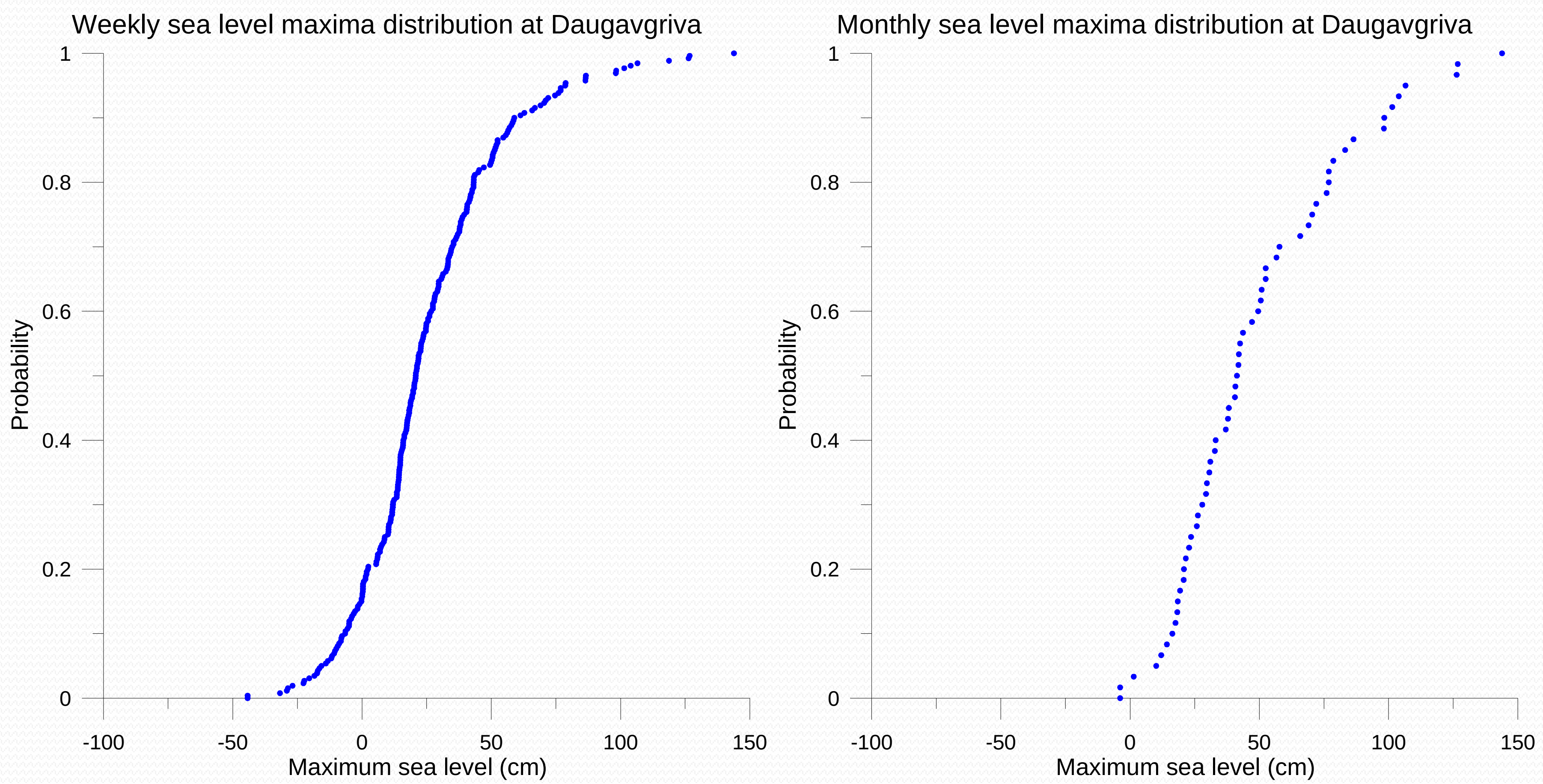
$$C(t) = C_0 + a + bt + c\sin\left(2\pi\frac{t}{T}\right) + d\cos\left(2\pi\frac{t}{T}\right)$$

At Daugavgriva:
C_0 = 500 cm
a = 5.3 cm
b = 6.3E-04 cm/d
c = -18.05 cm
d = 4.69 cm
r^2=0.28



Data and background information

- Maxima sea level distributions at Daugavgriva station:



Extreme sea level statistics- the theory

- **Principle I:** The sea level maximum x=max(X_1, X_2,...X_N), where X_i (i=1, 2, ..., N) are i.i.d., for $N\to\infty$ follows

  the General Extreme Value (GEV) distribution: $F(x)=\exp\left(-\left(1+\xi\frac{x-\mu}{\sigma}\right)^{-\frac{1}{\xi}}\right)$
  which reduces to:

a) reversed Weibull distribution, if $\xi<0$: $$F_W(x)=\begin{cases}\exp\left(-\left(\frac{\mu_W-x}{\sigma_W}\right)^{\alpha_W}\right), & x\le\mu_W\\ 1, & x>\mu_W\end{cases}$$

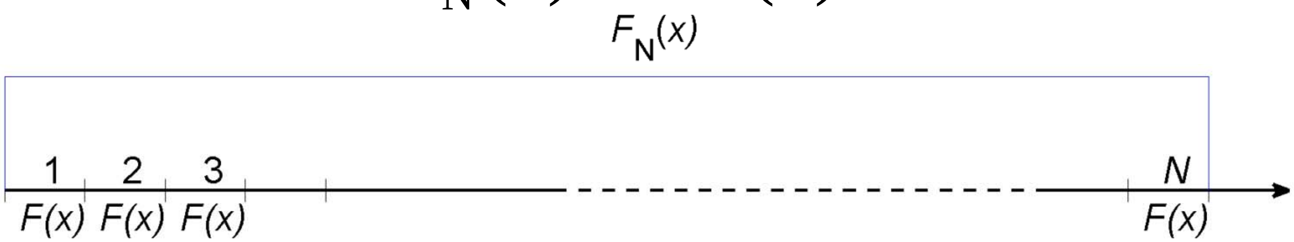
b) Gumbel distribution, if $\xi\to 0$: $$F_G(x)=\exp\left(-\exp\left(\frac{x-\mu_G}{\sigma_G}\right)\right)$$

c) Frechet distribution, if $\xi>0$: $$F_F(x)=\begin{cases}0, & x<\mu_F\\ \exp\left(-\left(\frac{x-\mu_F}{\sigma_F}\right)^{-\alpha_F}\right), & x\ge\mu_F\end{cases}$$

Extreme sea level statistics- the theory

- **Principle II**- the independence of maximum occurrences

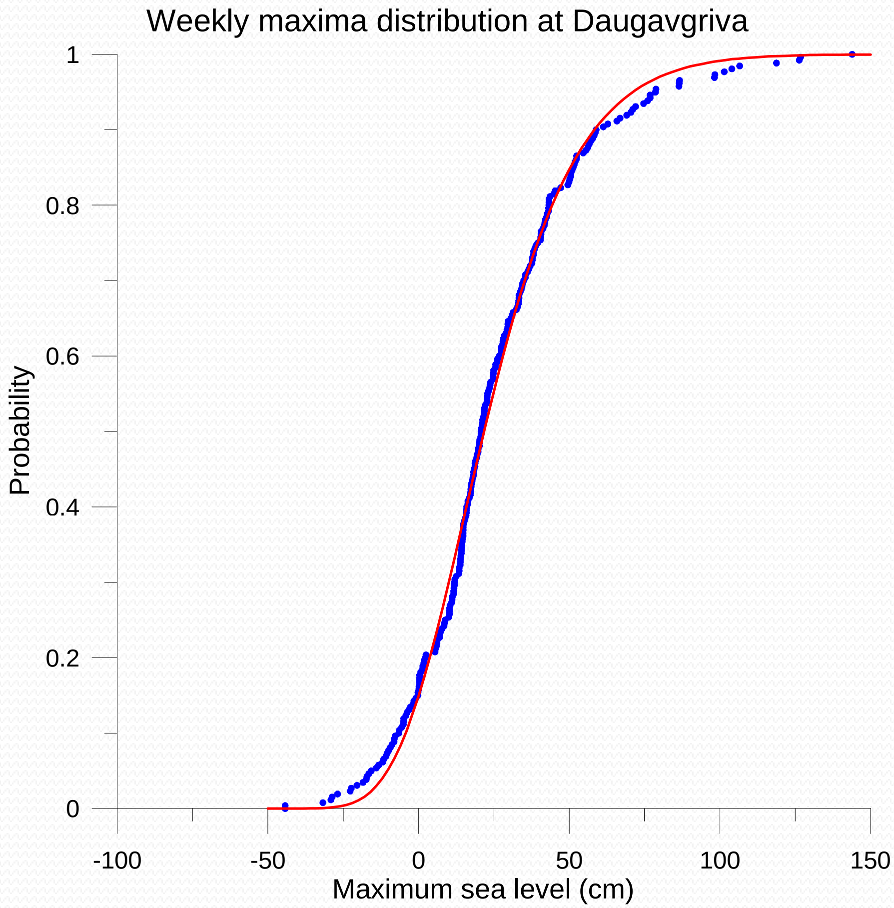
$$F_{\mathrm{N}}(x) = F^{N}(x)$$

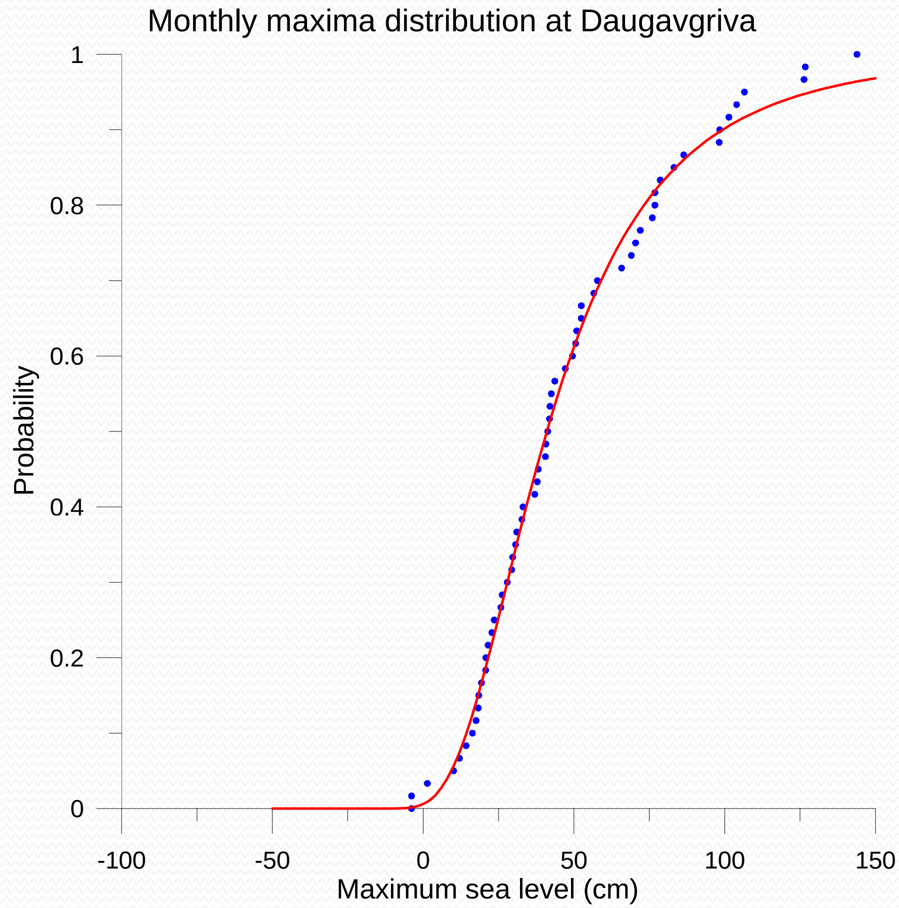


- The probability of occurrence of a maximum level *x* in *N* adjacent intervals must equal to the product of probabilities of *x* in each of the *N* intervals.

Extreme sea level statistics

- GEV distributions:

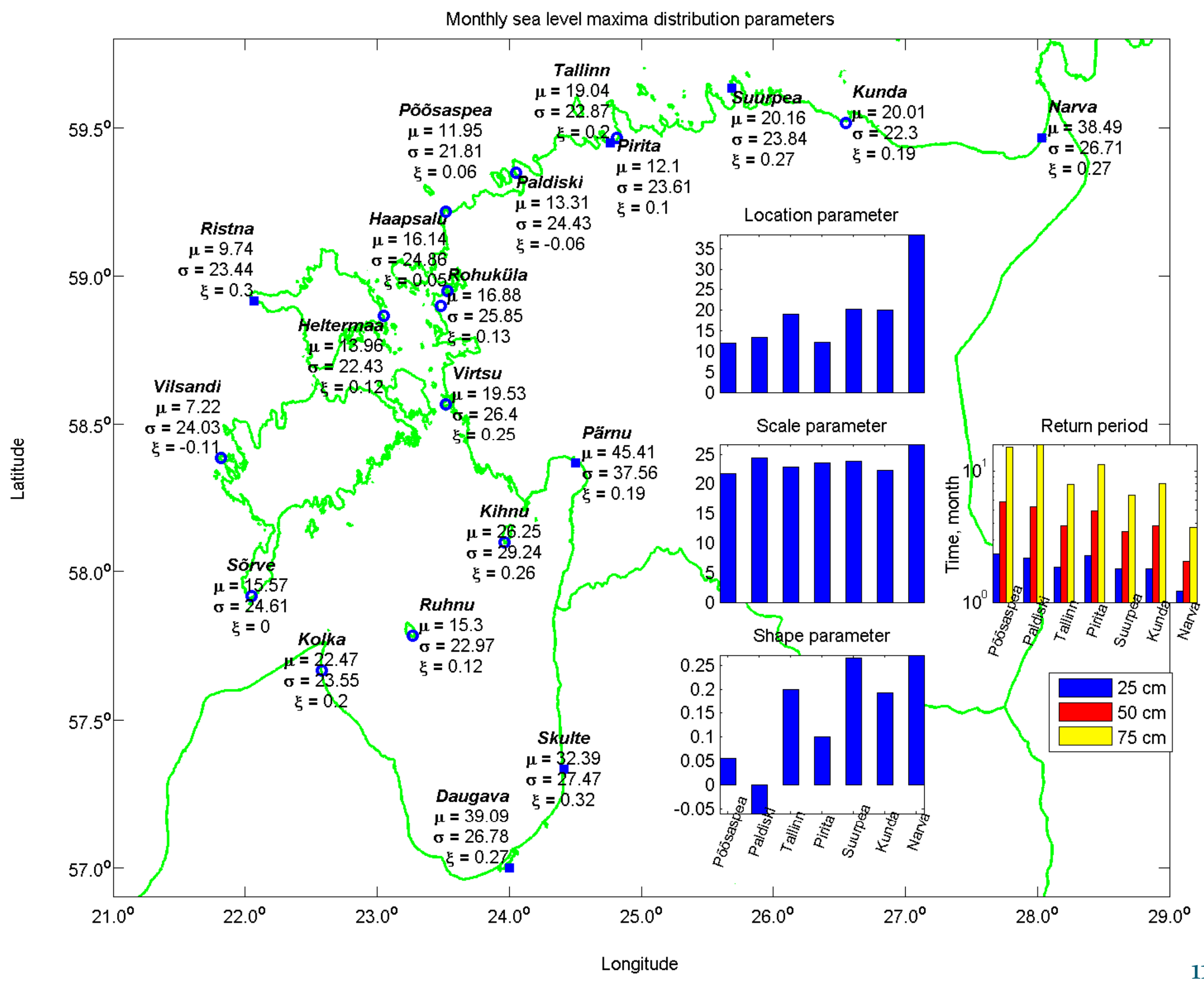




Extreme sea level statistics

- Approximated GEV distribution parameters:

There is tendency for easterly increase of approximated GEV parameters in the Gulf of Finland. The return period for various maximum sea levels reduces with the increase of parameters in the basin.

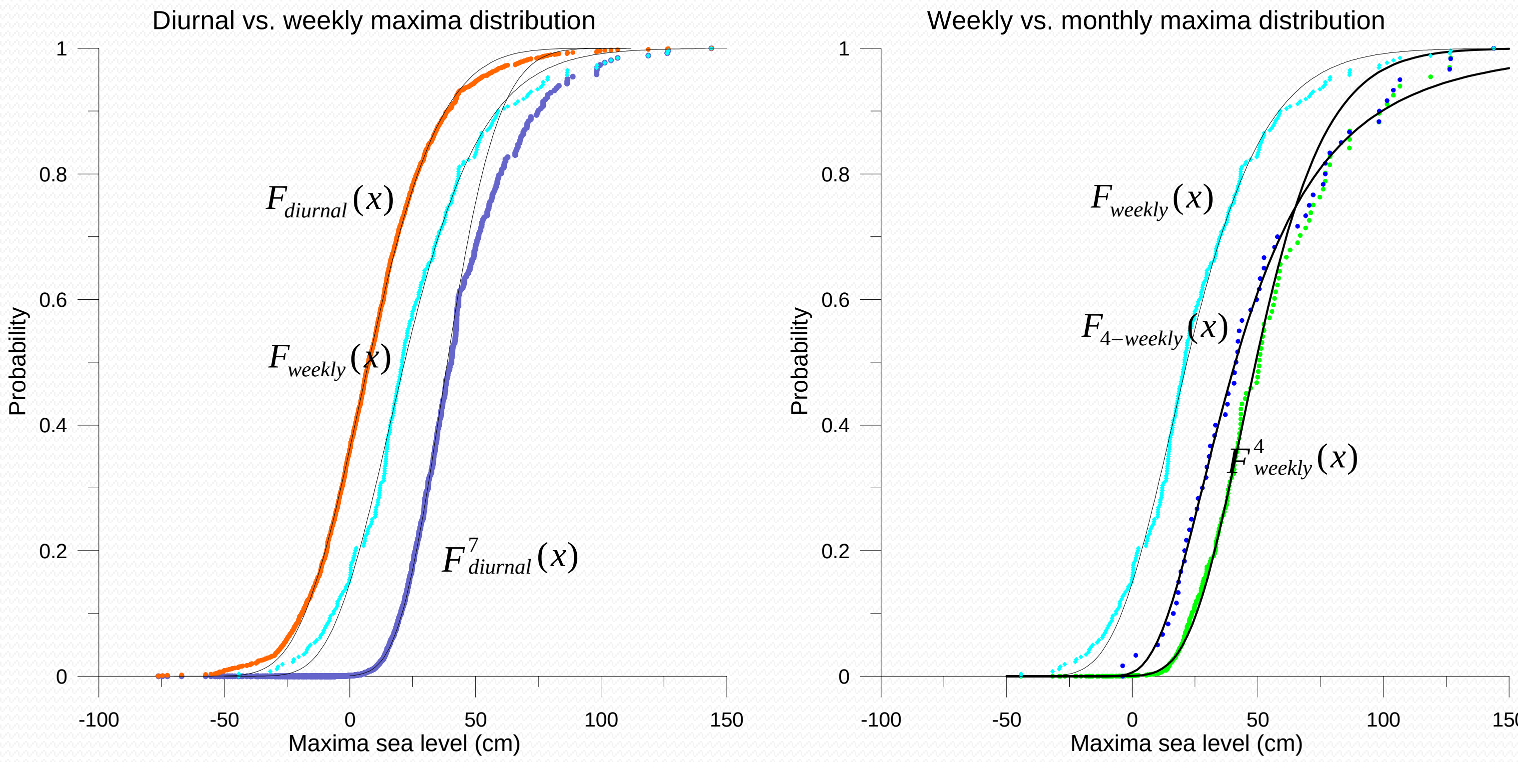


Extreme sea level statistics

- According to the **principle II** the 4-weekly and weekly maxima probability should equal the weekly and diurnal maxima probability to the power 4 and 7, respectively:

$$F_{weekly}^4(x) = F_{4-weekly}(x)$$

$$F_{diurnal}^7(x) = F_{weekly}(x)$$



Conclusion: Principle II is not met

$$F_{weekly}^4(x) \neq F_{4-weekly}(x)$$

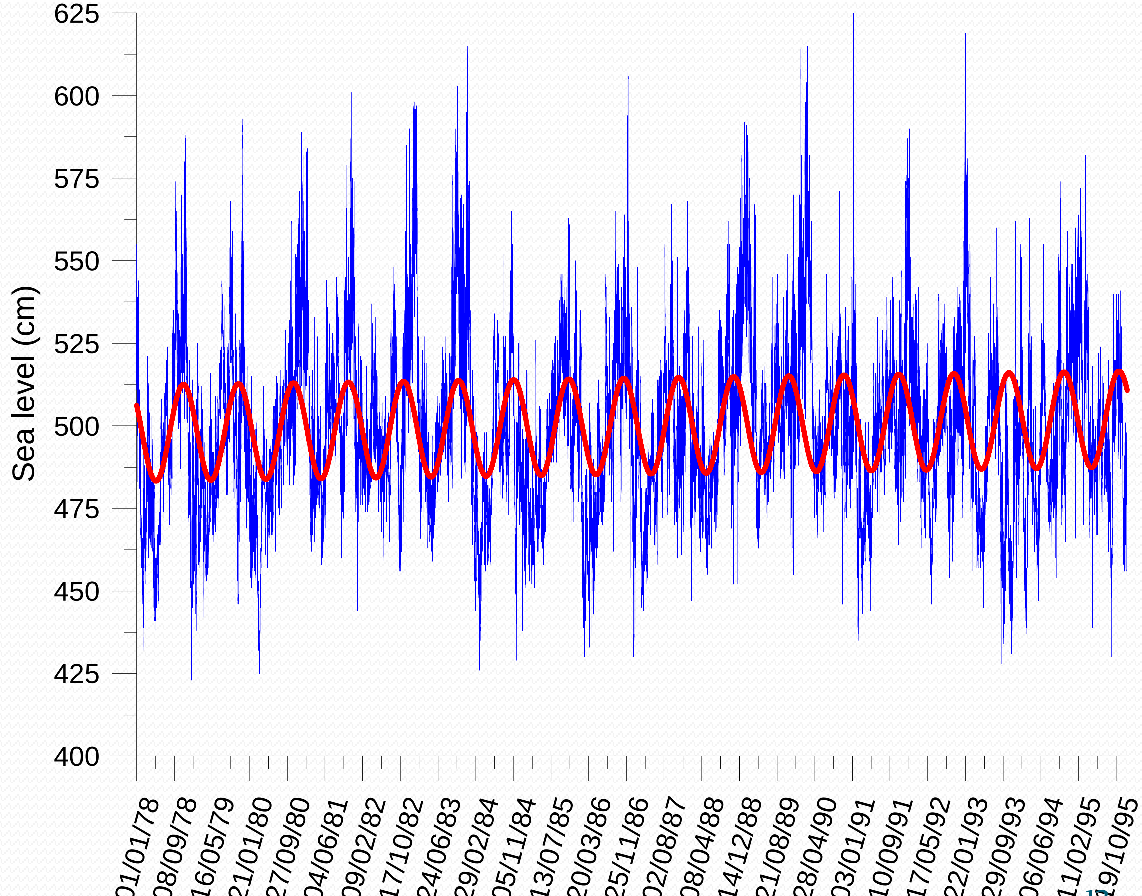
$$F_{diurnal}^7(x) \neq F_{weekly}(x)$$

Statistically independent maximum sea level events

- It seems that the maxima over shorter time intervals (day, week, month) appear statistically dependant $F_N(x) \neq F^N(x)$
- To check the statement we use 18 years long time-series measured at Tallinn station

Data number, *N*: 153512
Recording lag, Δt: 1 hour
Missing data number: 4264

Harmonics:
T= 365.25 d
C_0= 500 cm
a = -2.2 cm
b = 6.5E-04 cm/h
c = -11.83 cm
d = 8.50 cm
r^2 = 0.17



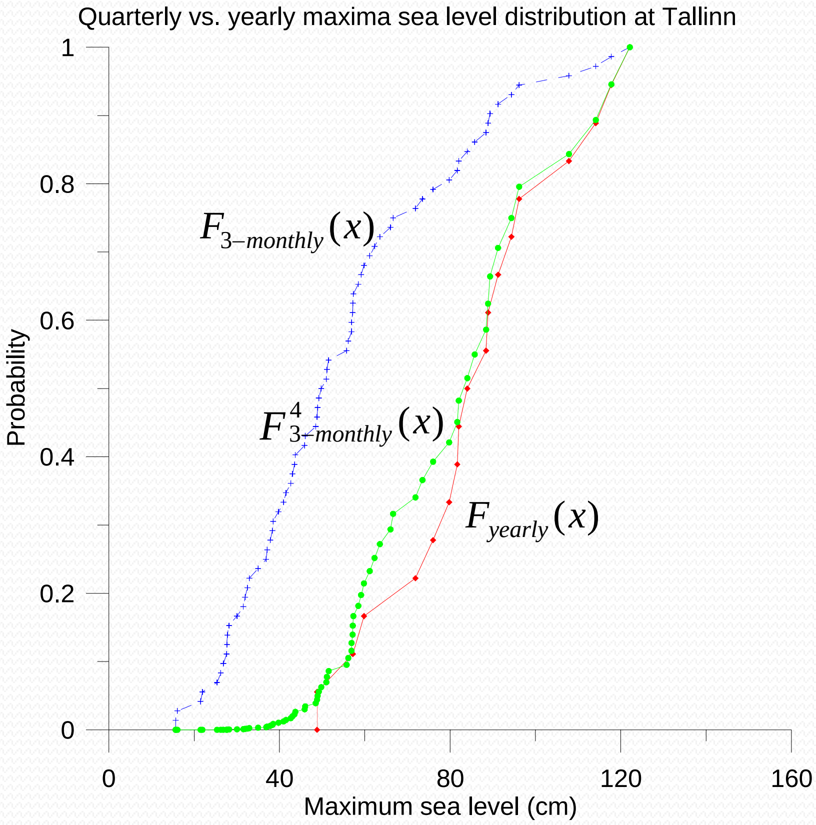
Statistically independent maximum sea level events

• Estimated distributions of the maximum sea level:

•Principle II is met if:

$$F^{4}_{3-monthly}(x) = F_{yearly}(x)$$

• Conclusion: principle II is met for the probabilities of the sea level maximum over longer (> quarter) time intervals.



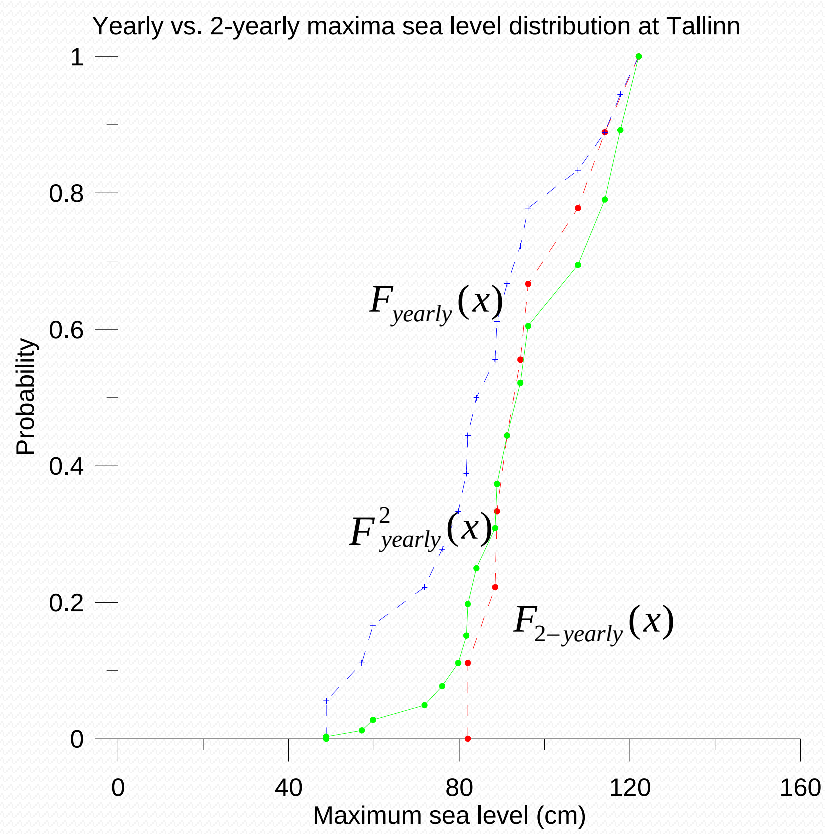
Statistically independent maximum sea level events

• Estimated distributions of the maximum sea level:

•Principle II is met if:

$$F^2_{yearly}(x) = F_{2-yearly}(x)$$

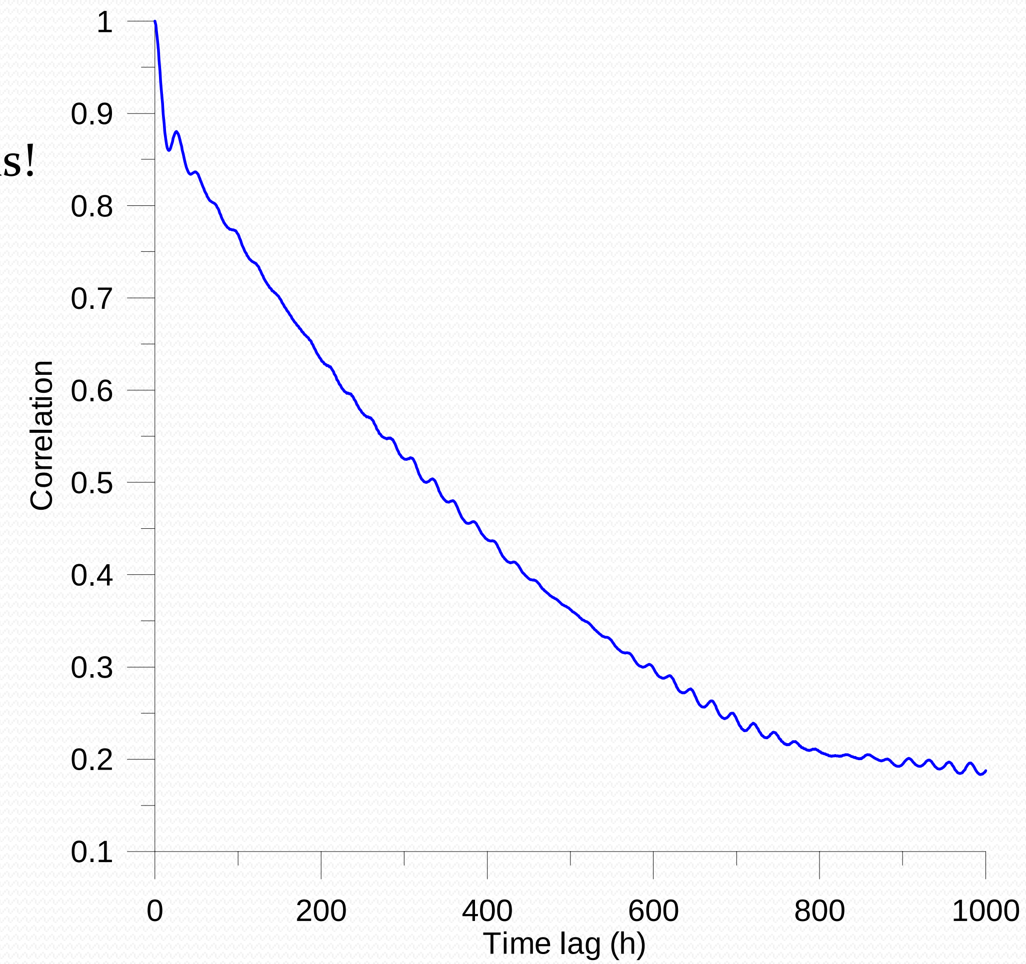
• Conclusion: The number of data is not sufficient for the maxima distribution to converge to the GEV. It can happen that the principle I is not met because of too small *N*.



Statistically independent maximum sea level events

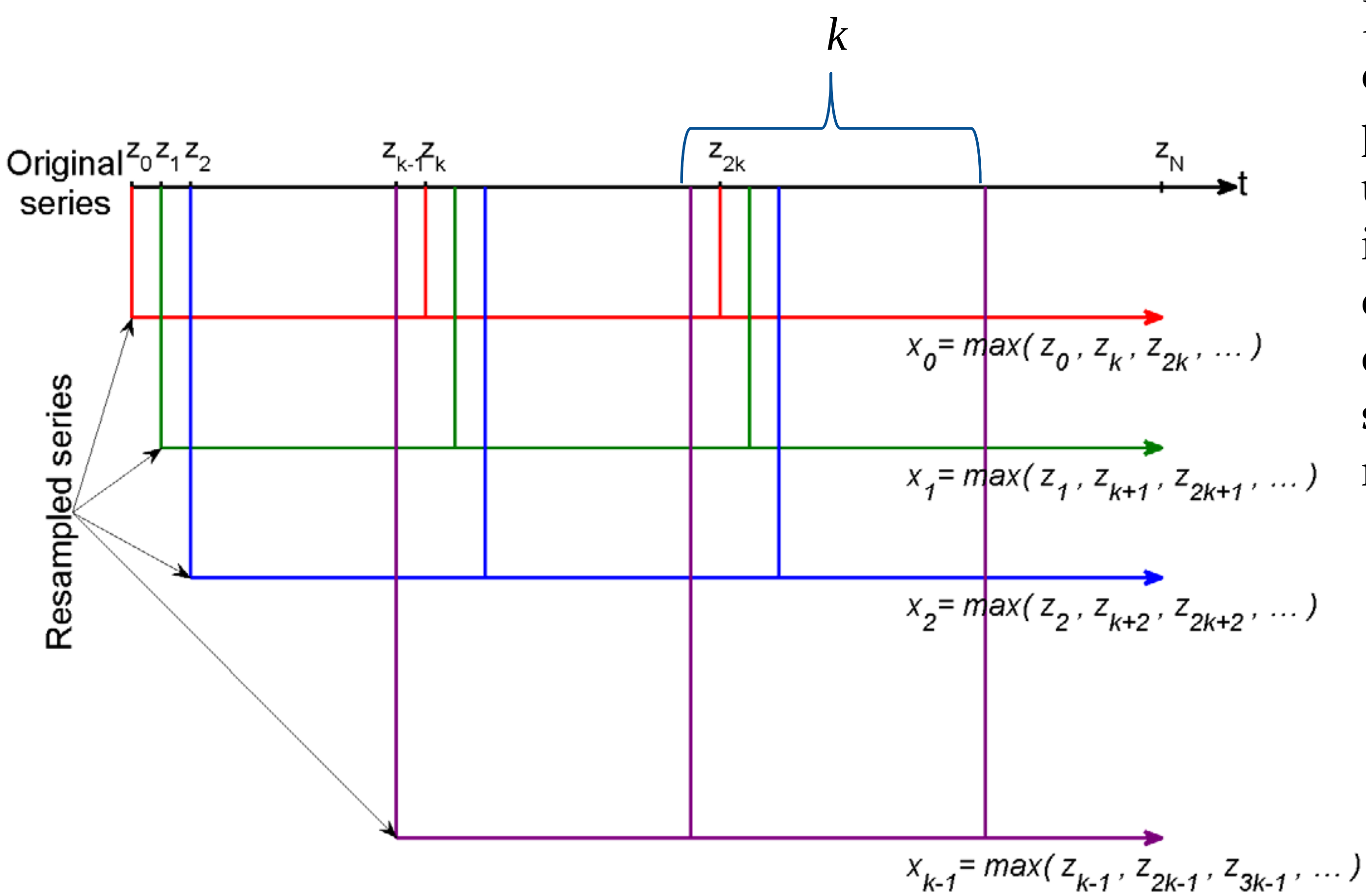
- Autocorrelation of the Tallinn series:

- The correlation scale is of the order of 10^3 h, or about 2 months!



Sea level maxima distributions- the model

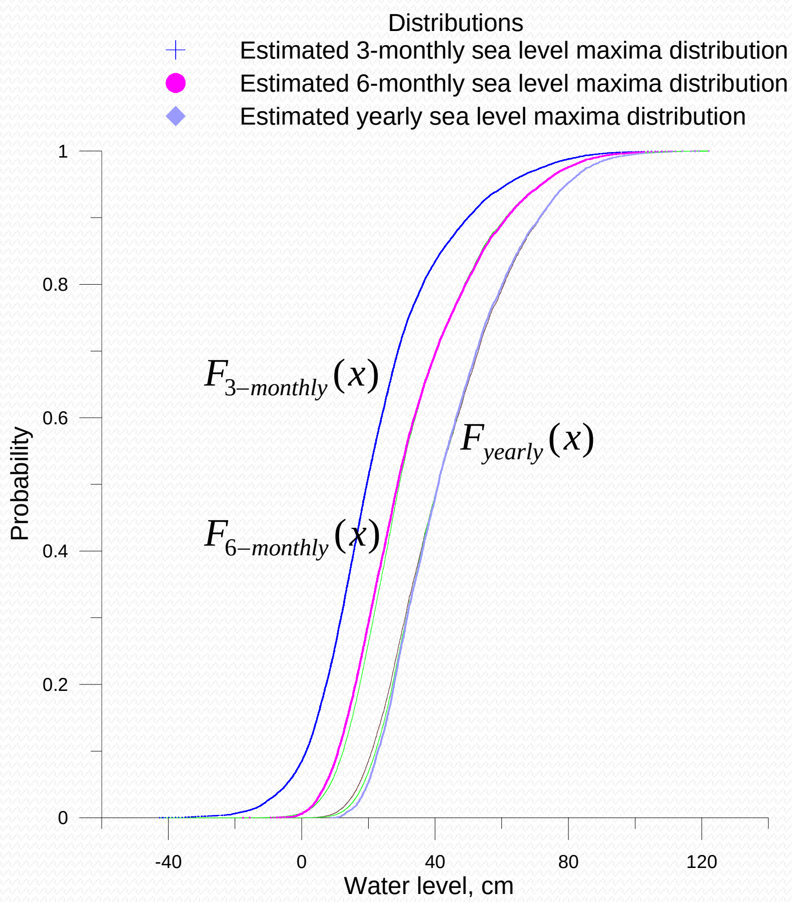
- Stochastic model:



Estimating distribution over period N: use resampling interval *k* to select elements from original data series to the resampled series.

Sea level maxima distributions- the model

- Maxima sea level distributions:



•Selected k was 438 h, with correlation 0.41

•Principle II is met if:

$$F_{3-monthly}^{2}(x) = F_{6-monthly}(x)$$

$$F_{3-monthly}^{4}(x) = F_{yearly}(x)$$

$$F_{6-monthly}^{2}(x) = F_{yearly}(x)$$

• Conclusion:

Principle I is met as the $N >> 1$

Sea level maxima distributions- the model

- Maxima sea level distributions:

▪ Selected k was 438

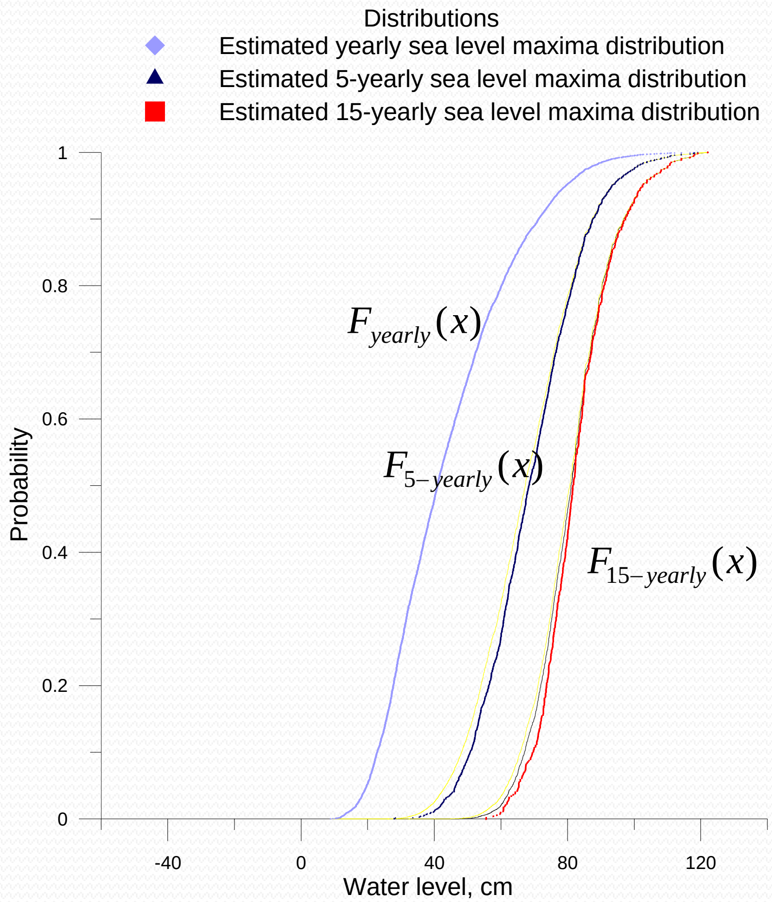
▪ Principle II is met if:

$$F^{5}_{yearly}(x) = F_{5-yearly}(x)$$

$$F^{15}_{yearly}(x) = F_{15-yearly}(x)$$

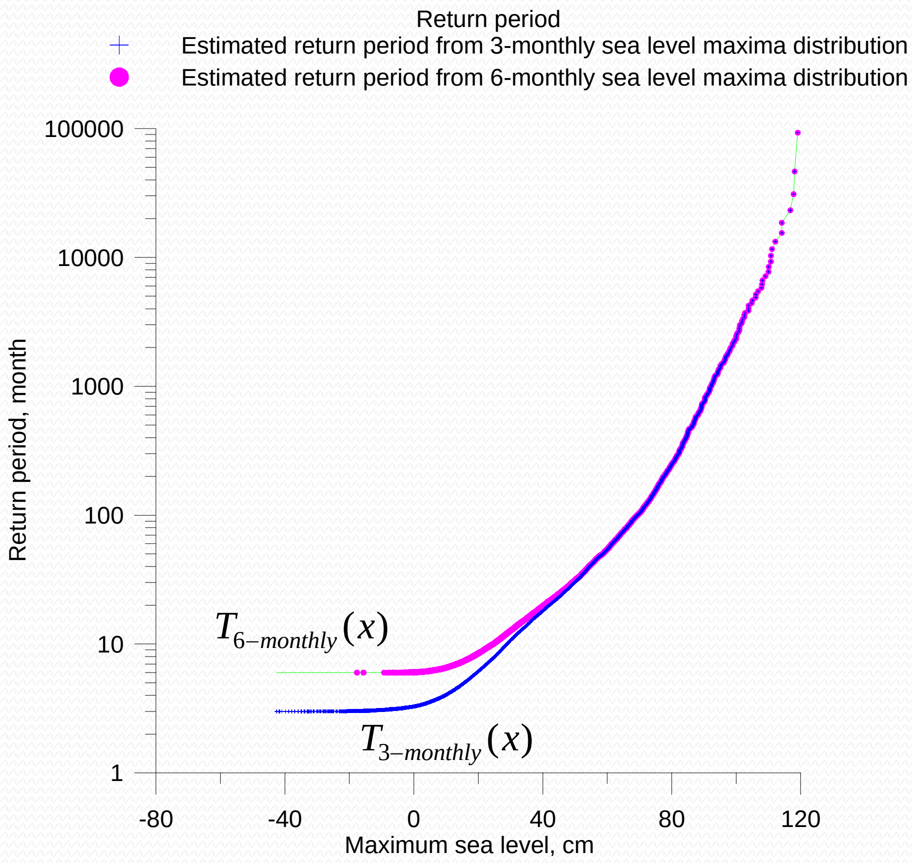
$$F^{3}_{5-yearly}(x) = F_{15-yearly}(x)$$

▪ Principle I is met



Sea level maxima distributions- the model

- Maxima sea level return periods:



- Return period was calculated:

$$T_{\mathrm{N}}(x) = \frac{T_0}{1 - F_{\mathrm{N}}(x)}$$

- T_0=3 months for 3-monthly maximum x
- T_0=6 months for 6-monthly maximum x

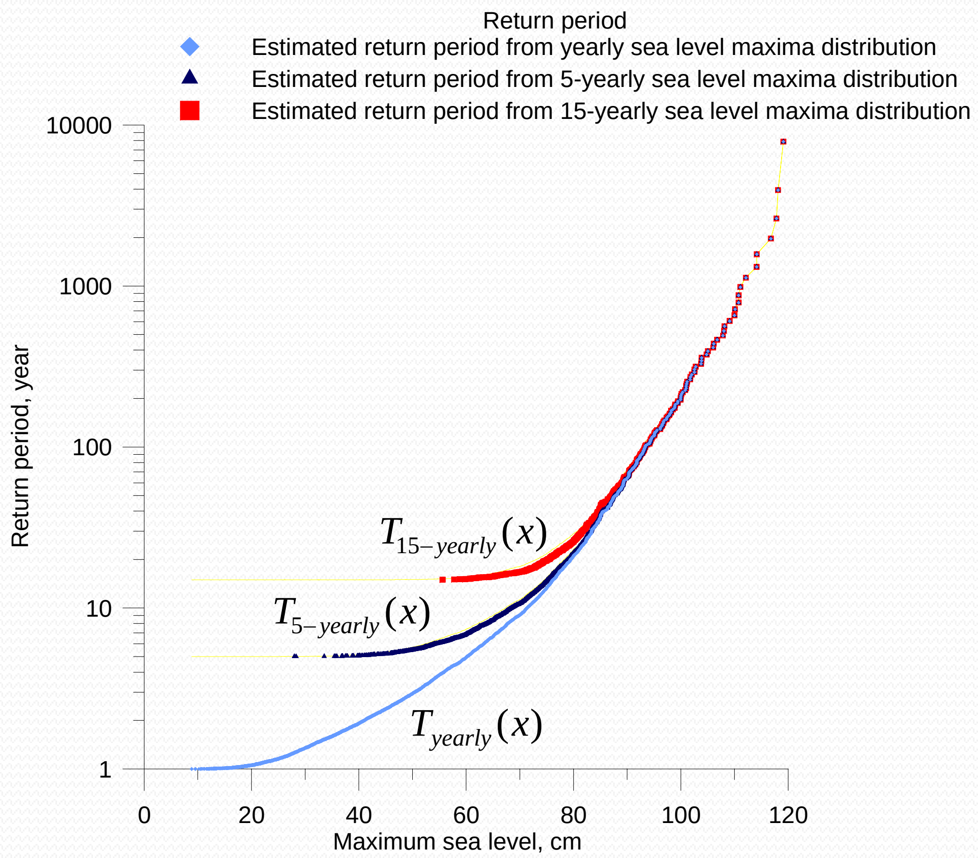
Sea level maxima distributions- the model

- Maxima sea level return periods:

- Return period was calculated:

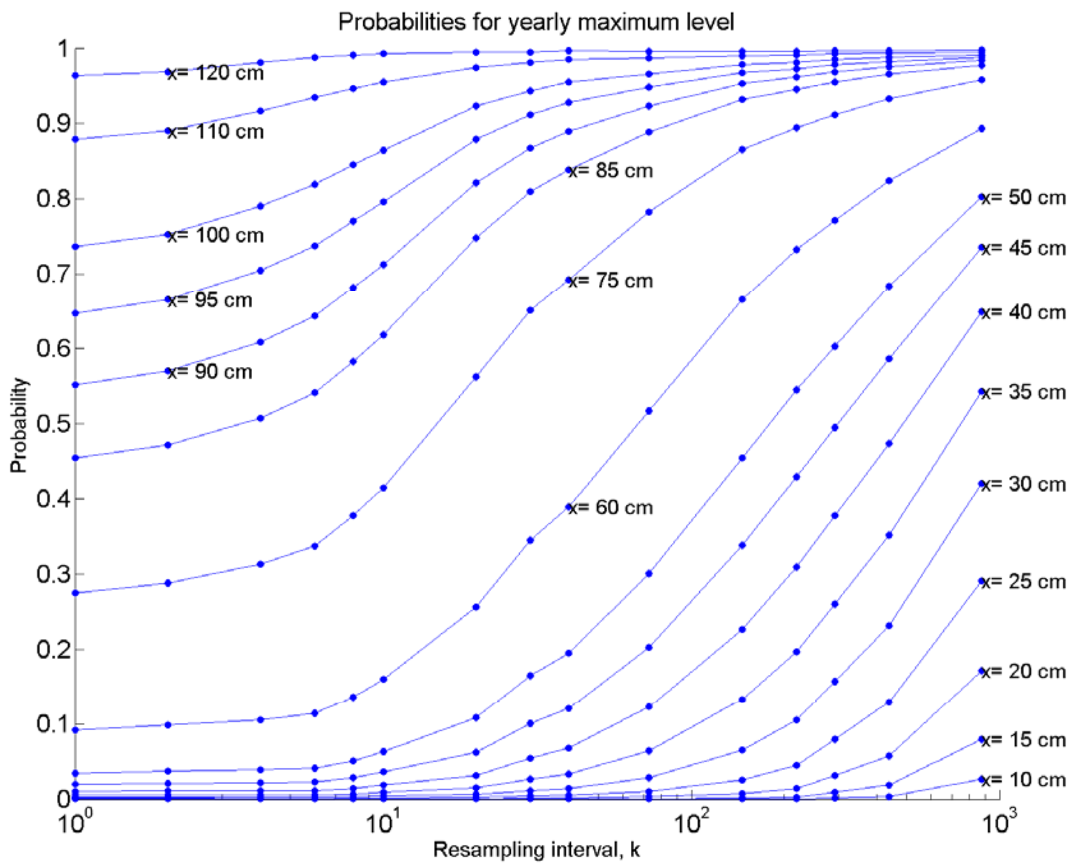
$$T_N(x) = \frac{T_0}{1 - F_N(x)}$$

- T_0=1 year
  for yearly maximum x
- T_o=5 years
  for 5-yearly maximum x
- T_o=15 years
  for 15-yearly maximum x



Sea level maxima distributions- the model

- Probabilities for various level vs. resampling interval:

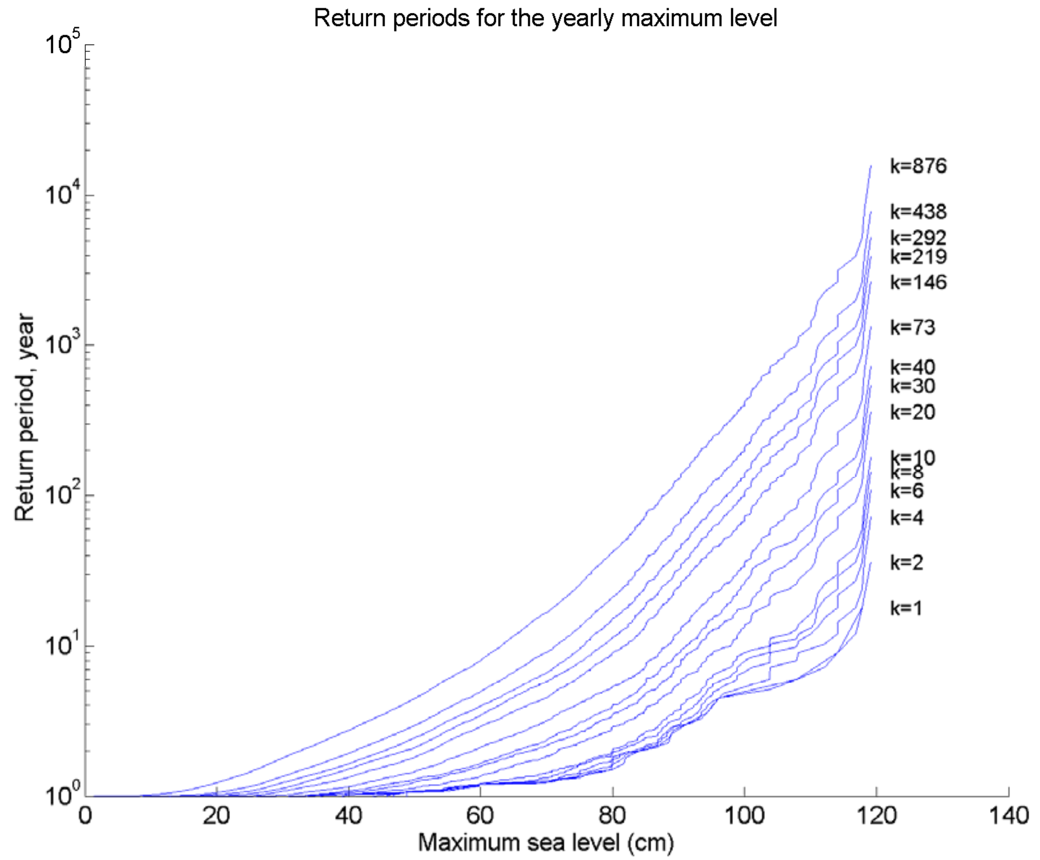


The yearly maximum sea level probability depends strongly on the resampling interval *k*.

Sea level maxima distributions- the model

- Return periods for various k:

  The return period depends on the resampling interval *k*.



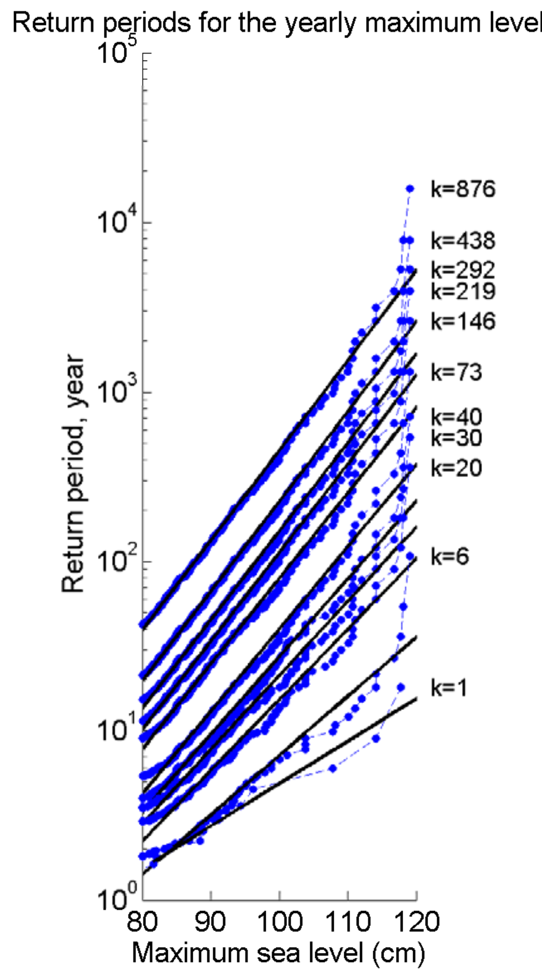
Sea level maxima distributions- the model

- Return periods for various k:
  - The return period for maximum sea level from 80 cm to 120 cm was modelled as :

$$\ln T = ax + b$$

- Parameters:

k	a	b	r^2
876	0.0530	-2.6494	0.9924
438	0.0530	-2.9507	0.9923
292	0.0523	-3.0531	0.9906
219	0.0523	-3.1791	0.9905
146	0.0505	-3.1478	0.9874
73	0.0486	-3.2631	0.9802
40	0.0459	-3.1550	0.9804
30	0.0438	-3.0502	0.9737
20	0.0417	-2.9855	0.9653
6	0.0350	-2.6468	0.9456
1	0.0250	-1.8117	0.9613



Conclusion

- Sea level maxima over time intervals shorter than quarter appear statistically dependent.
- Sea level maxima over intervals longer than 3 months, appear statistically independent.
- For the maxima over intervals longer than 3 months there is not sufficient number of data in our short data series to estimate their distributions- a new model has to be applied.
- The suggested statistical model accounts for the both indicated aspects explicitly, in the result of which the probability of maximum and respectively the return period appears to be dependent on the resampling interval k.
- The tests did not reveal any limit probability distribution within resampling intervals $k = 876 \ldots 1$ h.
- A proper relation between the conventional approach to sea level order statistics and our suggested model should be established.

Thank You for your attention!
Questions and comments please.